#1 a) 1) $\vec{EF}$ et $\vec{CO}$; $\vec{OP}$, $\vec{MN}$ et $\vec{KL}$ GH Mechants bons problème : VECTEURS

2) $\vec{EF}$ et $\vec{CO}$; $\vec{OP}$ et $\vec{KL}$

#2- $\vec{v} = (1,4)$ $\vec{w} = (-3,-2)$

#3- a) $\Delta x = 80 \cos 120 = -40$
$\Delta y = 80 \sin 120 = 69{,}28$

b) $\Delta x = 400 \cos 250 = -136{,}81$
$\Delta y = 400 \sin 250 = -375{,}88$

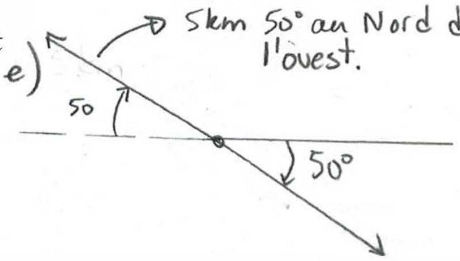
c) $\Delta x = 0$
$\Delta y = -3$

#4 a) $\vec{v}_1 + \vec{v}_2 = (5,2) + (1,-3) = (6,-1)$

b) $\vec{v}_1 + \vec{v}_2 + \vec{v}_3 = (5,2) + (1,-3) + (-3,2) = (3,1)$

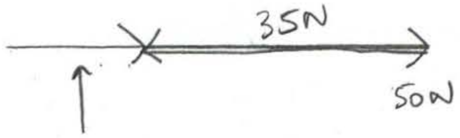
#5 a) vecteurs colineaires, opposés ou linéairement dependants
b) " orthogonaux
c) " lin. independants
d) voir a)
e) vecteurs colineaires ou lin. dépendants.
f) vecteurs equipollents ou colineairs ou lin. dependants

#6 a) $-\vec{v}$ b) $-\vec{AB}$ ou $\vec{BA}$ c) $-\vec{u} = (-3,2)$ d) 3cm S30 ouest e) 5km 50° au Nord de l'ouest.



#7 a) $\vec{AB} - \vec{AD} = \vec{AB} + \vec{DA} = \vec{DA} + \vec{AB} = \vec{DB}$
b) $\vec{BA} - \vec{DA} = \vec{BA} + \vec{AD} = \vec{BD}$
c) $\vec{AC} + \vec{AD} - \vec{AD} = \vec{AC}$!
d) $\vec{AC} + \vec{CD} + \vec{DB} = \vec{AB}$
e) $\vec{CD} - \vec{AD} + \vec{AB} = \vec{CD} + \vec{DA} + \vec{AB} = \vec{CB}$
f) $\vec{AD} - \vec{CD} - \vec{BC} = \vec{AD} + \vec{DC} + \vec{CB} = \vec{AB}$

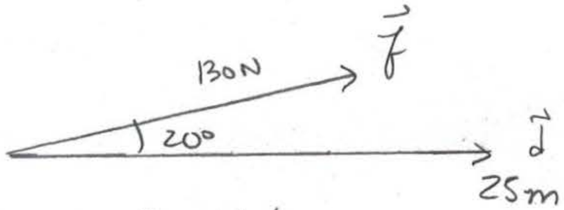
#8



$50N - 35N = 15N$

#9 $W = \|\vec{f}\|\,\|\vec{d}\|\cos\theta = 500 \cdot 500\cos 60 = 125\ 000$ Joules

#10



$W = 130 \cdot 25\cos 20° = 3054$ Joules.

#11 a) $3(2\vec{a} + 3\vec{a}) = 3 \cdot 5\vec{a} = 15\vec{a} = 15(-2,3) = \underline{\underline{(-30,45)}}$

b) $(2\vec{a} \cdot 3\vec{b})\,\vec{a}$

$[2(-2,3) \cdot 3(-1,-5)] \cdot (-2,3)$

$[(-4,6) \cdot (-3,-15)] \cdot (-2,3)$

$(12 + -90) \cdot (-2,3)$

$-78\ (-2,3) = \underline{\underline{(156,-234)}}$

c) $(\vec{a} + 2\vec{b}) - 2(\vec{a} + \vec{b})$

$= \vec{a} + 2\vec{b} - 2\vec{a} - 2\vec{b}$

$= -\vec{a} = \underline{\underline{(2,-3)}}$

d) $(\vec{a} \cdot \vec{b})(\vec{a} + \vec{b}) + 2\vec{b} - 3\vec{a}$

$[(-2,3) \cdot (-1,-5)]\,[(-2,3) + (-1,-5)] + 2(-1,-5) - 3(-2,3)$

$(2 + -15)\ (-3,-2) + \underbrace{(-2,-10) + (6,-9)}$

$-13\ (-3,-2) + (4,-19)$

$(39,26) + (4,-19)$

$\underline{\underline{(43,7)}}$

#12 a) $\vec{w} = (-2, 3)$

$\vec{w} = a\vec{u} + b\vec{v}$

$(-2,3) = a(2,-1) + b(-1,3)$

$(-2,3) = (2a, -a) + (-b, 3b)$

$(-2,3) = (2a-b, -a+3b)$

$$\begin{array}{rl} 2a - b = -2 \Rightarrow & 2a - b = -2 \\ 2(-a+3b = 3) \Rightarrow & + \; -2a + 6b = 6 \\ \hline & 5b = 4 \end{array}$$

$b = 0{,}8 \Rightarrow a = ?$ $\quad 2a - 0{,}8 = -2$

$a = -0{,}6$

$\vec{w} = -0{,}6\vec{u} + 0{,}8\vec{v}$

b) $\vec{w} = (3,-4)$

$\vec{w} = a\vec{u} + b\vec{v}$

$(3,-4) = \underbrace{a(3,-4) + b(-1,3)}$

⋮ voir #12a)

$(3,-4) = (2a-b, -a+3b)$

$$\begin{array}{rl} 2a - b = 3 \rightarrow & 2a - b = 3 \\ 2 \cdot (-a+3b = -4) \rightarrow & + \; -2a + 6b = -8 \\ \hline & 5b = -5 \end{array}$$

$b = -1 \Rightarrow a = ?$ $\quad 2a + 1 = 3$

$a = 1$

$\vec{w} = \vec{u} - \vec{v}$

#13 $\vec{u} = (2,3)$ $\quad \frac{\Delta y}{\Delta x}$ de $\vec{u} = \frac{3}{2}$

$\vec{v} = (4,6)$ $\quad \frac{\Delta y}{\Delta x}$ de $\vec{v} = \frac{6}{4} = \frac{3}{2}$

> ils sont // donc NON car ils doivent être lin. indépendants

#14 $\vec{V} = a\vec{s} + b\vec{r}$

$(3,4) = a(3,1) + b(2,4)$
$(3,4) = (3a,a) + (2b,4b)$
$(3,4) = (3a+2b, a+4b)$

$3a+2b = 3 \rightarrow 3a+2b = 3$
$3(a+4b=4) \rightarrow \underline{-\;3a+12b = 12}$

$-10b = -9$

$b = 0,9$

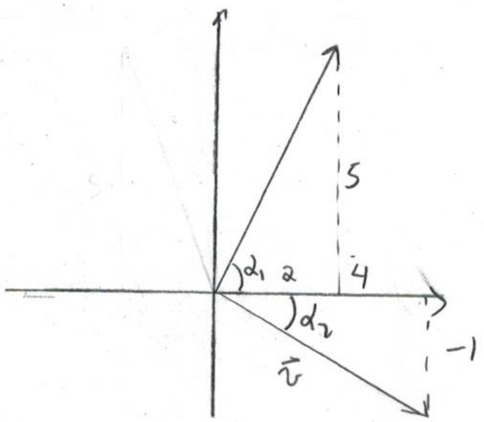
$a + 4(0.9) = 4$

⋮

$a = 0,4$

$\vec{V} = 0,4\vec{s} + 0,9\vec{r}$

#15 $\vec{u} = (2,5)$ et $\vec{v} = (4,-1)$



$\alpha_1 = \tan^{-1}\frac{5}{2} = 68,2$

$\alpha_2 = \tan^{-1}\frac{1}{4} = 14,04$

$\theta = 68,2 + 14,04 = \underline{\underline{82,24^\circ}}$

$\boxed{\text{ou}}$ $\vec{u} \cdot \vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$

$2\times4 + 5\times(-1) = \sqrt{2^2+5^2}\,\sqrt{4^2+1^2}\cos\theta$

$3 = \sqrt{29}\,\sqrt{17}\cos\theta$

$0,13... = \cos\theta$

$\underline{\underline{\theta = 82,23^\circ}}$

#16 $\vec{F}_1 : (-185, 0)$

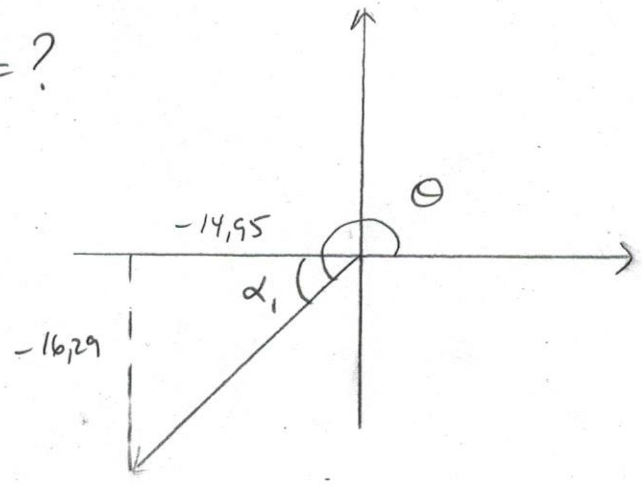
$\vec{F}_2 :$ $\left.\begin{array}{l} \Delta x = 90 \cos 22 = 83{,}45 \\ \Delta y = 90 \sin 22 = 33{,}71 \end{array}\right\} (83{,}45,\ 33{,}71)$

$F_3 :$ $\left.\begin{array}{l} \Delta x = 100 \cos 330° = 86{,}60 \\ \Delta y = 100 \sin 330° = -50 \end{array}\right\} (86{,}6,\ -50)$

Résultante $= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (-14{,}95,\ -16{,}29)$

$$\|\vec{F}_1 + \vec{F}_2 + \vec{F}_3\| = \sqrt{(-14{,}95)^2 + (-16{,}29)^2} = \underline{22{,}11\text{N}}$$

$\theta = ?$



$\alpha_1 = \tan^{-1} \dfrac{16{,}29}{14{,}95}$

$= 47{,}46°$

$\theta = 180 + 47{,}46 = 227{,}46°$

#17 $\vec{v} = (c, 8)$ $\vec{u} = (4, 3)$

$\dfrac{\Delta y}{\Delta x}$ de $\vec{u}$: $\dfrac{3}{4} \xrightarrow{\perp} -\dfrac{4}{3} = \dfrac{\Delta y}{\Delta x}$ de $\vec{v}$

$-\dfrac{4}{3} = \dfrac{8}{c} \Rightarrow \underline{c = -6}$

ou

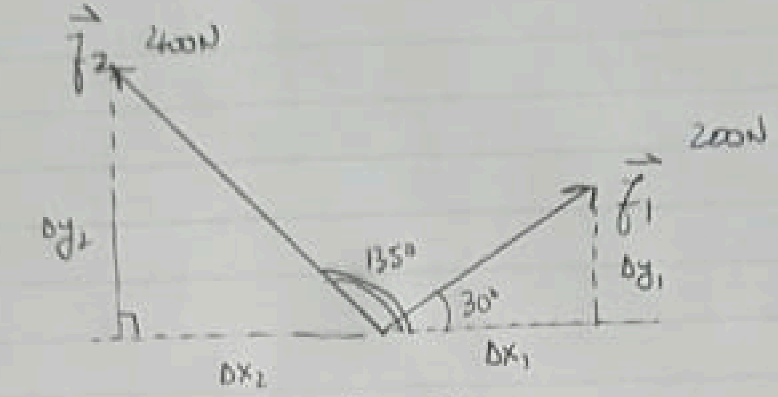
$\vec{u} \cdot \vec{v} = 0$ si $\perp$

$(4,3) \cdot (c, 8) = 0$

$4c + 24 = 0$

$\underline{c = -6}$

\# 18



$\vec{f}_1$: $\Delta x_1 = 200 \cos 30° = 173{,}21$
$\Delta y_1 = 200 \sin 30° = 100$

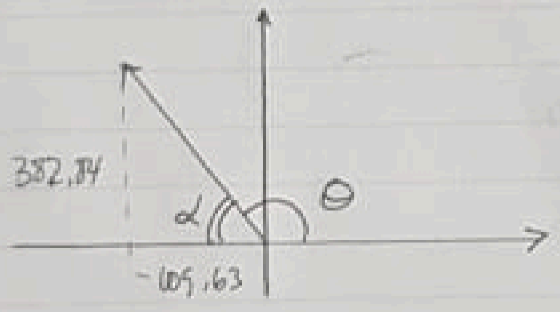
$\vec{f}_2$: $\Delta x_2 = 400 \cos 135° = -282{,}84$
$\Delta y_2 = 400 \sin 135° = 282{,}84$

a) résultante : $\Delta x = 173{,}21 + -282{,}84 = -109{,}63$
$\Delta y = 100 + 282{,}84 = 382{,}84$

$\vec{r} = (-109{,}63 , 382{,}84)$

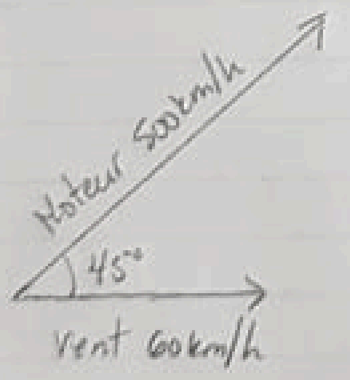
$\|\vec{r}\| = \sqrt{109{,}63^2 + 382{,}81^2} = \boxed{358{,}23N}$

b)



$\theta = 180 - \alpha$
$\theta = 180 - \tan^{-1}\left(\frac{382{,}84}{109{,}63}\right) = 180 - 74{,}02$
$= \boxed{105{,}58°}$

\#19



Moteur : $\Delta x = 500 \cos 45 = 353{,}55$
$\Delta y = 500 \sin 45 = 353{,}55$

Vent : $\Delta x = 60$
$\Delta y = 0$

$\vec{r} = (413{,}55 , 353{,}55)$

$\|\vec{r}\| = \underline{544{,}08 \text{ km/h}}$

\#20 $\vec{r} = 1\vec{g} - 2\vec{p}$

#21 $\vec{u} = (18,-12)$
$\vec{v} = (\Delta x, -4)$

$\left.\begin{array}{l}\vec{u}=(18,-12)\\ \vec{v}=(\Delta x,-4)\end{array}\right\}$ Si $\vec{u} \parallel \vec{v} \Rightarrow$ pentes sont égales $\left.\begin{array}{l} \\ \dfrac{-12}{18} = \dfrac{-4}{\Delta x} \quad \Delta x = 6\end{array}\right\}$ $\underline{\vec{v} = (6,-4)}$

$\vec{w} = (0,-10)$ car orienté SUD

$\vec{x} = \vec{v} - \vec{w} = (6,-4) - (0,-10) = \underline{\underline{(6,6)}}$

$\vec{y} = (18,8) = a(18,-12) + b(6,6)$

$(18,8) = (18a, -12a) + (6b, 6b)$

$(18,8) = (18a + 6b, -12a + 6b)$

$$\begin{array}{rl} & 18a + 6b = 18 \\ - & -12a + 6b = 8 \\ \hline & 30a = 10 \end{array}$$

$\underline{a = \frac{1}{3}}$ $\quad b = ?$ $\quad 18(\frac{1}{3}) + 6b = 18$

$\underline{b = 2}$

$$\vec{y} = \frac{1}{3}\vec{u} + 2\vec{x}$$

#22 Si $\vec{VQ} \perp \vec{QT} \Rightarrow$ produit de leurs pentes $= -1$

$\frac{3}{1} \cdot \frac{\Delta y}{-12} = -1 \Rightarrow \frac{3\Delta y}{-12} = -1 \Rightarrow \Delta y = 4$ $\Big\}$ $\underline{\vec{QT} = (-12,4)}$

$\vec{MT} = (5\sqrt{2}\cos 45, 5\sqrt{2}\sin 45) = \underline{\underline{(5,5)}}$

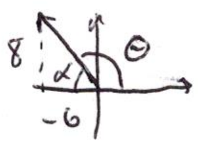
$\vec{HV} = (8 - -2, 1 - -5) = \underline{\underline{(10,6)}}$

$\vec{HM} =$ résultante $= \vec{HV} + \vec{VQ} + \vec{QT} + \vec{TM}$

$= \vec{HV} + \vec{VQ} + \vec{QT} + (-\vec{MT})$

$= (10,6) + (1,3) + (-12,4) + (-5,-5) = \boxed{(-6,8)}$

$\|\vec{HM}\| = \sqrt{(-6)^2 + 8^2} = \boxed{10\text{ km}}$ $\quad \theta_{\vec{HM}}$



$\alpha = \tan^{-1}\frac{8}{6} = 53{,}13°$

$\theta = 180 - \alpha = \boxed{126{,}87°}$

#23 $\vec{u} = (192 \cos 36{,}87^\circ, 192 \sin 36{,}87^\circ)$

$\vec{u} = (153{,}6, 115{,}2)$

$\vec{v} = (240 \cos 126{,}87, 240 \sin 126{,}87)$ $\quad \theta_{\vec{v}} = 180 - 53{,}13^\circ$ *

$= (-144, 192)$

$\vec{r}_{esultante} = (1200 \cos 73{,}7398^\circ, 1200 \sin 73{,}7398^\circ)$

$= (336, 1152)$

$\vec{r} = \boxed{a}\,\vec{u} + \boxed{b}\,\vec{v} = ($combien de $\vec{u}$ + combien de $\vec{v})$

force ouvrier → $\vec{u}$; force esclave → $\vec{v}$

$(336, 1152) = a(153{,}6, 115{,}2) + b(-144, 192)$

$(336, 1152) = (153{,}6a, 115{,}2a) + (-144b, 192b)$

$(336, 1152) = (153{,}6a - 144b, 115{,}2a + 192b)$

$1{,}\overline{3} \cdot (153{,}6a - 144b = 336) \Rightarrow 204{,}8a - 192b = 448$

$115{,}2a + 192b = 1152 \Rightarrow 115{,}2a + 192b = 1152$

$320a = 1600$

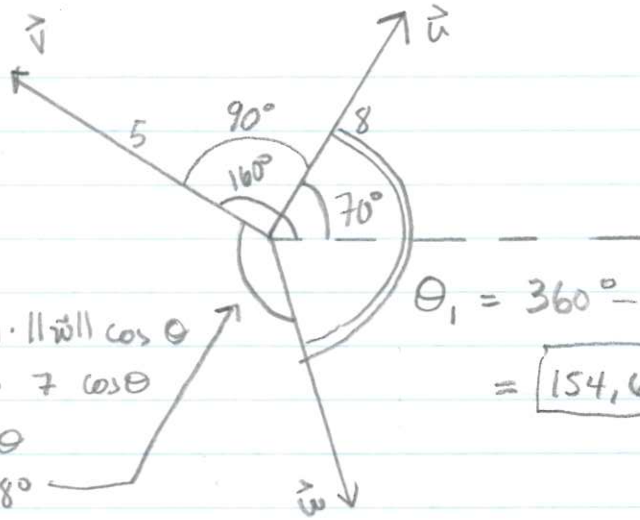
$\boxed{\begin{array}{l} a = 5 \text{ ouvrier} \\ b = 3 \text{ esclaves} \end{array}}$

$\Rightarrow 115{,}2(5) + 192b = 1152$

$\downarrow$

$b = 3$

#24



$\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cos\theta$

$-15 = 5 \cdot 7 \cos\theta$

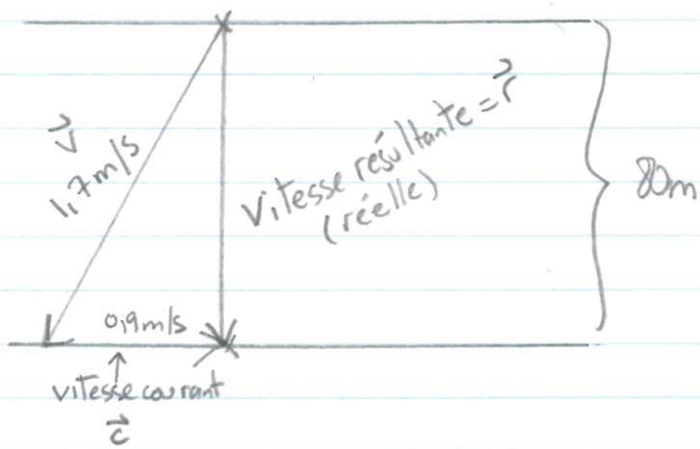
$-0,428\ldots = \cos\theta$

$\theta = 115,38°$

$\theta_1 = 360° - 90 - 115,38°$

$= \boxed{154,62°}$

#25



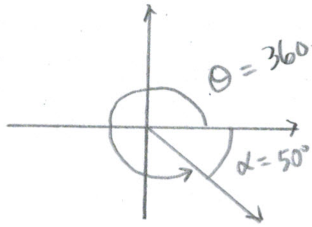
$\|\vec{r}\|^2 + 0,9^2 = 1,7^2$: pythagore

$\|\vec{r}\| = 1,44$ m/s

temps $= 80\text{m} \div 1,4\text{m/s} = \boxed{55,47 \text{ sec}}$

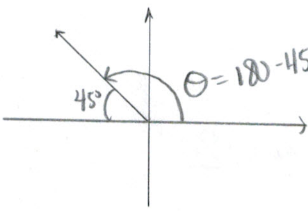
#26 c) 18 cm²

#27



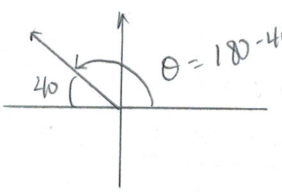
$\theta = 360 - 50° = 310°$

#28



$\theta = 180 - 45° = 135°$

#29



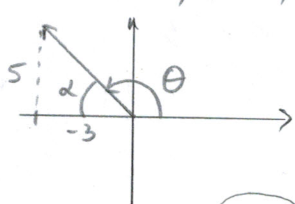
$\theta = 180 - 40 = 140°$

#30 $\vec{AB} = (-3 - -11, 8 - 4) = (8, 4)$

#31 $\|\vec{v}\| = \sqrt{(-12)^2 + 5^2} = \sqrt{169} = 13$

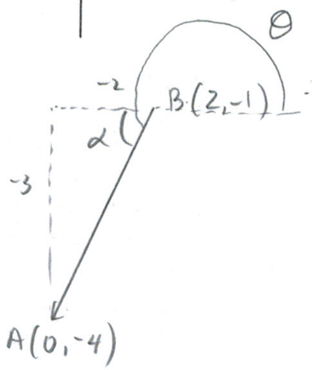
#32 $\vec{AB} = (5 - -3, 4 - -2) = (8, 6) \Rightarrow \|\vec{AB}\| = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$

#33



$\alpha = \tan^{-1}\frac{5}{3} = 59,04 \quad \theta = 180 - 59,04 = 120,96°$

#34 a)

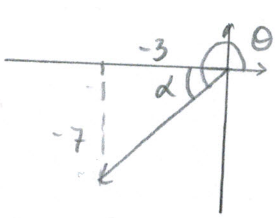


$\vec{BA} = (0-2, -4 - -1) = (-2, -3)$

$\alpha = \tan^{-1}\frac{3}{2} = 56,31$

$\theta = 180 + 56,31 = 236,31°$

b) $\vec{AB} = (3, 7) \quad -\vec{AB} = (-3, -7)$



$\alpha = \tan^{-1}\frac{7}{3} = 66,80$

$\theta = 180 + 66,80 = 246,80$

#35 a) $\vec{AB} - \vec{CB} - \vec{FC} = \vec{AB} + \vec{BC} + \vec{CF} = \vec{AF}$

b) $\vec{AB} + \vec{BD} + \vec{DC} - \vec{AC} = \vec{AB} + \vec{BD} + \vec{DC} + \vec{CA} = \vec{AA} = \vec{O}$

#36 a) $\vec{u} + \vec{v} = (-4,3) + (-2,-1) = (-6,2)$

b) $\vec{u} + \vec{v} = (5,-8) + (-3,2) = (2,-6)$

#37 $\vec{r} = -3(-2,7) = (6,-21)$

#38 $\vec{r} = -\frac{1}{2}(-8,0) = (4,0)$

#39 $\vec{u} \cdot \vec{v} = (2,1) \cdot (-2,4) = -4 + 4 = 0$ donc les vecteurs sont orthogonaux: $\underline{90°}$

#40 $\vec{c} = a\vec{a} + b\vec{b}$

$(-1,4) = a(-3,-4) + b(5,4)$

$(-1,4) = (-3a,-4a) + (5b,4b)$

$(-1,4) = (-3a+5b, -4a+4b)$

$4(-3a + 5b = -1) \rightarrow -12a + 20b = -4$

$3(-4a + 4b = 4) \rightarrow -12a + 12b = 12$

$8b = -16$

$b = -2$

$-3a + 5(-2) = -1$

$-3a - 10 = -1$

$-3a = 9$

$a = -3$

$$\boxed{\vec{c} = -3\vec{a} - 2\vec{b}}$$

#41 Le vecteur $\vec{d}$ car il n'est pas parallèle avec le $\vec{a}$