

Mechants bons problèmes : Les FONCTIONS RÉELLES

#1 a) $\text{dom } f = [-8, 2]$ b) $\text{Ima } f = [-4, 5, 3]$ c) zéros: $x = -6, 5$
 $x = -4$
 $x = 1$

d) Max absolu $y = 3$ Min absolu $y = -4, 5$

e) Max relatif $y = 2, 5$ Min relatif $y = -4$

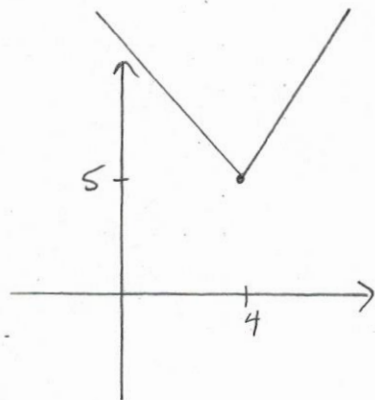
#2 a) $a = 3, b = -2, h = 5, k = 7$ b) $a = 6, b = 0, 5, h = -4, k = -9$

#3 a) $(g \circ f)(x) = g(f(x)) = 2(-3x) - 3 = -6x - 3$

b) $(h \circ g)(x) = h(g(x)) = -6(2x - 3) + 2 = -12x + 20$

#4 $y = \frac{2}{3} |-3(x-4)| + 5$

$y = 2|x-4| + 5$



#5 a) $|x-4| - 1 = -2x + 5$

$|x-4| = -2x + 6$



$x-4 \geq 0$
 $x \geq 4$

$x-4 < 0$
 $x < 4$

$x-4 = -2x+6$

$3x = 10$

~~$x = \frac{10}{3}$~~

$x-4 = 2x-6$

$-x = -2$

$x = 2$

b) $2|x-2| + 2x = 0$

$2|x-2| = -2x$

$|x-2| = -x$



$x-2 \geq 0$
 $x \geq 2$

$x-2 < 0$
 $x < 2$

$x-2 = -x$

$2x = 2$

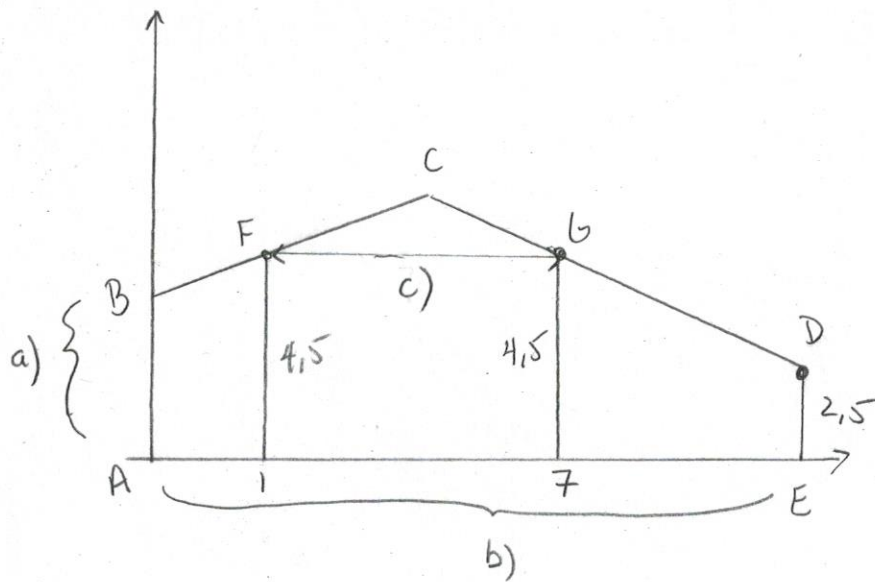
~~$x = 1$~~

$x-2 = x$

$0 = 2$

\emptyset

#6



a) Ordonnée à l'origine $x=0$ $y=?$ $y = -\frac{1}{2}|0-4|+6$ $y = \underline{4m}$

b) $x_D=?$ $2,5 = -\frac{1}{2}|x-4|+6$

$y_D=2,5$ $7 = |x-4|$



$x-4 > 0$ $x-4 < 0$

$x > 4$ $x < 4$

$x-4 = 7$ $x-4 = -7$

$x = 11$ $x = -3$

rep: 11m

c) $x=?$ sig $y=4,5$

$4,5 = -\frac{1}{2}|x-4|+6$

$-1,5 = -\frac{1}{2}|x-4|$

$3 = |x-4|$



$x-4 > 0$ $x-4 < 0$

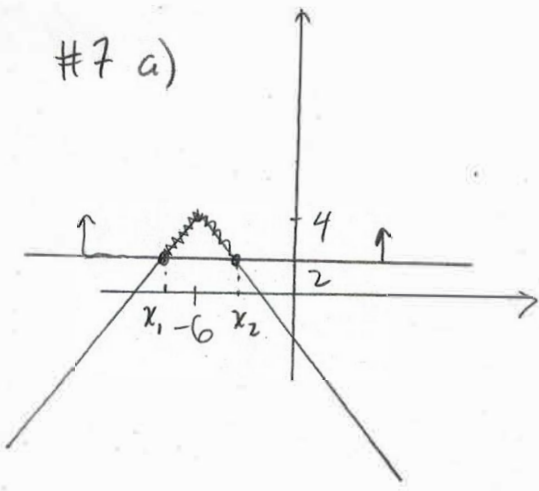
$x > 4$ $x < 4$

$x-4 = 3$ $x-4 = -3$

$x = 7$ $x = 1$

6m

#7 a)



$$x = ? \text{ si } y = 2$$

$$2 = -2|x+6| + 4$$

$$1 = |x+6|$$

$$\oplus \quad \ominus$$

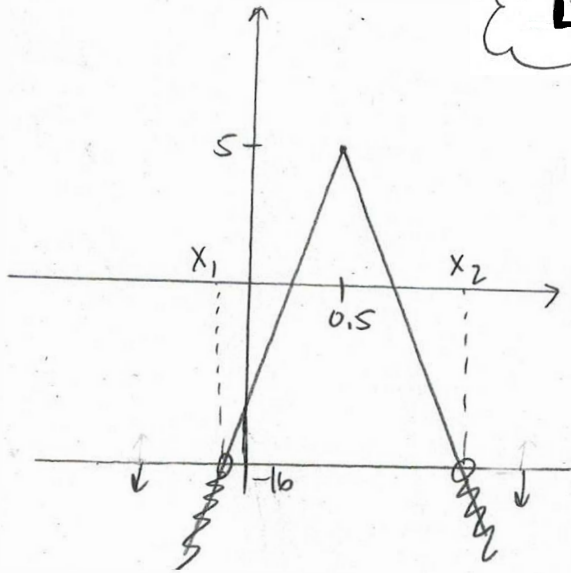
$$x \geq -6 \quad x < -6$$

$$x+6 = 1 \quad x+6 = -1$$

$$x = -5 \quad x = -7$$

$$[-7, -5]$$

b)



$$y = -3|2(x-0,5)| + 5$$

$$y = -6|x-0,5| + 5$$

$$x = ? \text{ si } y = -16$$

$$-16 = -6|x-0,5| + 5$$

$$3,5 = |x-0,5|$$

$$\oplus \quad \ominus$$

$$x \geq 0,5 \quad x < 0,5$$

$$x-0,5 = 3,5 \quad x-0,5 = -3,5$$

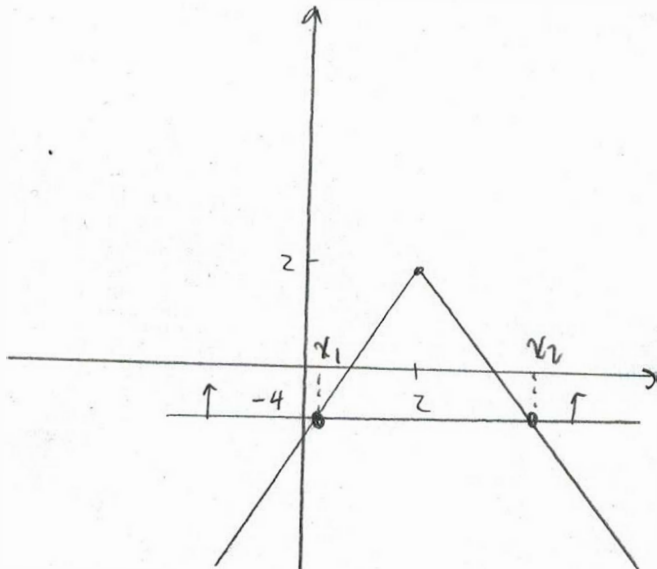
$$x = 4$$

$$x = -3$$

$$(-\infty, -3] \cup [4, +\infty)$$

$$c) y = -2|2(x-2)| + 2$$

$$y = -4|x-2| + 2$$



$$x = ? \text{ si } y = -4$$

$$-4 = -4|x-2| + 2$$

$$1,5 = |x-2|$$

$$\oplus \quad \ominus$$

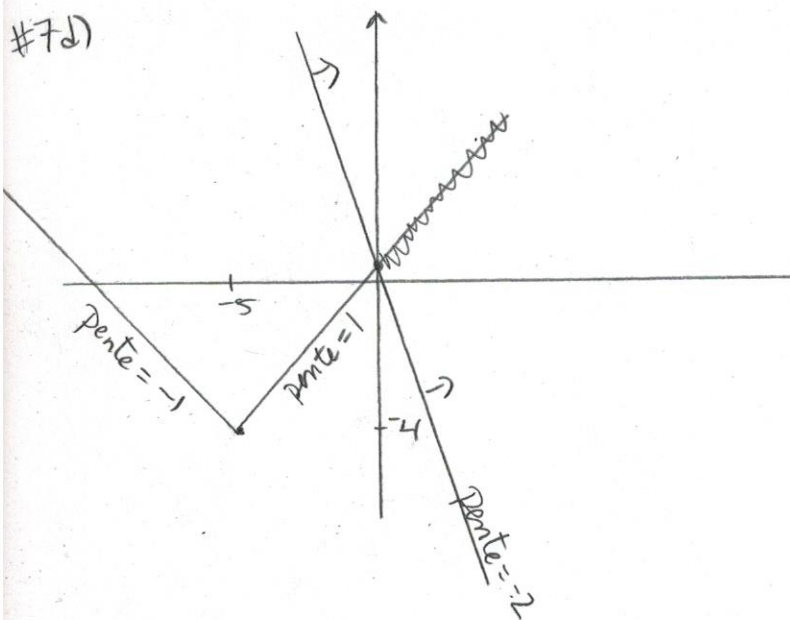
$$x \geq 2 \quad x < 2$$

$$x-2 = 1,5 \quad x-2 = -1,5$$

$$x = 3,5 \quad x = 0,5$$

$$[0,5, 3,5]$$

#7d)



$$|x+5| - 4 = -2x + 1$$

$$|x+5| = -2x + 5$$

⊕ / ⊖

$$x \geq -5 \quad x < -5$$

$$x+5 = -2x+5$$

$$3x = 0$$

$$x = 0$$

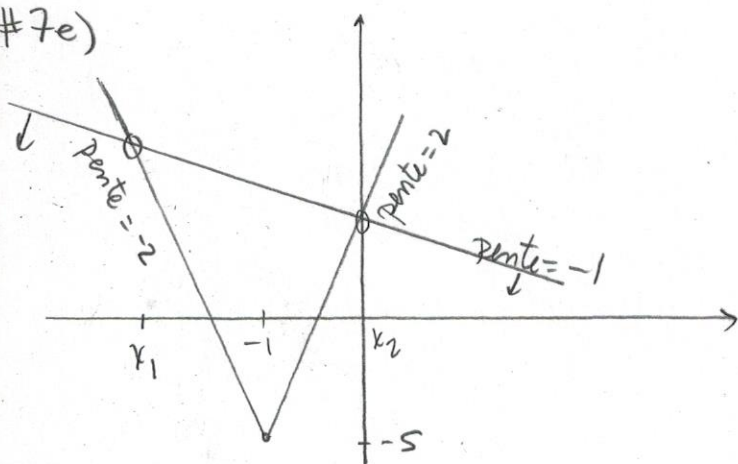
$$x+5 = 2x-5$$

$$-x = -10$$

$$x = 10$$

$[0, +\infty)$

#7e)



$$2|x+1| - 5 = -x - 3$$

$$2|x+1| = -x + 2$$

$$|x+1| = -0,5x + 1$$

⊕ / ⊖

$$x \geq -1 \quad x < -1$$

$$x+1 = -0,5x+1$$

$$1,5x = 0$$

$$x = 0$$

$$x+1 = 0,5x - 1$$

$$0,5x = -2$$

$$x = -4$$

$] -4, 0 [$

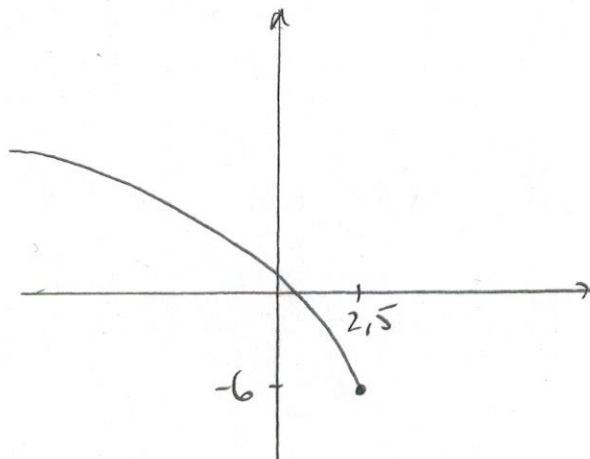
#8 $y = 2\sqrt{-2x+5} - 6$

a) $y = 2\sqrt{-2(x-2,5)} - 6$

b) $a=2 \quad b=-2 \quad h=2,5 \quad k=-6$

c) $\text{dom } f =]-\infty, 2,5]$

$\text{Ima } f = [-6, +\infty)$



#9 a) $2\sqrt{x+1} - 1 = 4x - 9$

$2\sqrt{x+1} = 4x - 8$

$\sqrt{x+1} = 2x - 4$ $\begin{array}{l} x+1 \geq 0 \\ x \geq -1 \end{array} \quad \left| \quad \begin{array}{l} 2x-4 \geq 0 \\ x \geq 2 \end{array} \right.$

$x+1 = (2x-4)^2$

$x+1 = 4x^2 - 16x + 16$

$0 = 4x^2 - 17x + 15$

$\frac{17 \pm \sqrt{289 - 240}}{8}$

$x_1 = 3$
 $x_2 = 1,25$ car

b) $4\sqrt{2x-3} = 12 - x$

$\sqrt{2x-3} = 3 - 0,25x$ $\begin{array}{l} 2x-3 \geq 0 \\ x \geq 1,5 \end{array} \quad \left| \quad \begin{array}{l} 3-0,25x \geq 0 \\ -0,25x \geq -3 \\ x \leq 12 \end{array} \right.$

$2x-3 = (3-0,25x)^2$

$2x-3 = 9 - 1,5x + 0,0625x^2$

$0 = 0,0625x^2 - 3,5x + 12$

$\frac{3,5 \pm \sqrt{12,25 - 3}}{0,125}$

$x_1 = 52,33$ à rejeter
 $x_2 = 3,67$

c) $3\sqrt{-3x+6} - 5 = 0,5x - 1$

$3\sqrt{-3x+6} = 0,5x + 4$

$\sqrt{-3x+6} = \frac{0,5x+4}{3}$

$\begin{array}{l} -3x+6 \geq 0 \\ x \leq 2 \end{array} \quad \begin{array}{l} \frac{0,5x+4}{3} \geq 0 \\ x \geq -8 \end{array}$

$\sqrt{-3x+6} = \left(\frac{0,5x+4}{3}\right)^2$

$-3x+6 = \frac{0,25x^2 + 4x + 16}{9}$

$-27x + 54 = 0,25x^2 + 4x + 16$

$0 = 0,25x^2 + 31x - 38$

$\frac{-31 \pm \sqrt{961 + 38}}{0,5}$

$x_1 = 1,21$
 $x_2 = -125,21$ car

10 $A_1(t) = A_2(t)$

$$18t + 300 = 90\sqrt{t-4} + 300$$

$$18t = 90\sqrt{t-4}$$

$$0,2t = \sqrt{t-4} \quad \begin{array}{l|l} t-4 \geq 0 & 0,2t \geq 0 \\ t \geq 4 & t \geq 0 \end{array}$$

$$0,04t^2 = t - 4$$

$$0,04t^2 - t + 4 = 0$$

$$\frac{t \pm \sqrt{1 - 0,16t}}{0,08} \quad \begin{array}{l} x_1 = 20 \\ x_2 = 5 \end{array}$$

À 5 min, et à 20 min.

11 a) $y = 4\sqrt{9x+27} + 6$

$$x = 4\sqrt{9y+27} + 6$$

$$\frac{x-6}{4} = \sqrt{9y+27} \quad \begin{array}{l|l} \frac{x-6}{4} \geq 0 & 9y+27 \geq 0 \\ x \geq 6 & y \geq -3 \end{array}$$

$$\frac{(x-6)^2}{16} = 9y+27$$

$$\frac{(x-6)^2}{16} - 27 = \frac{9y}{9}$$

$$\boxed{\frac{(x-6)^2}{144} - 3 = y}$$

b) $x = 4\sqrt{-(y-2)} - 8$

$$\frac{x+8}{4} = \sqrt{-(y-2)} \quad \begin{array}{l|l} \frac{x+8}{4} \geq 0 & -(y-2) \geq 0 \\ x \geq -8 & y-2 \leq 0 \\ & y \leq 2 \end{array}$$

$$\frac{(x+8)^2}{16} = -(y-2)$$

$$\frac{(x+8)^2}{-16} = y-2 \Rightarrow \boxed{\frac{(x+8)^2}{-16} + 2 = y}$$

#12 a) $y = \frac{300}{x} + 8$ x : nb d'élèves
 y : coût/élève

b) $x=0$ $y=8$

c) coût minimum = $k = 8$ \$

#13 a) x : altitude en km
 y : pression en kpa

$x=0$ $y=?$ $y = \frac{530}{0+5} = \underline{\underline{106 \text{ kpa}}}$

b) Si $y = \frac{106}{2} = 53$ $x=?$

$53 = \frac{530}{x+5}$

$53(x+5) = 530$

$53x + 265 = 530$

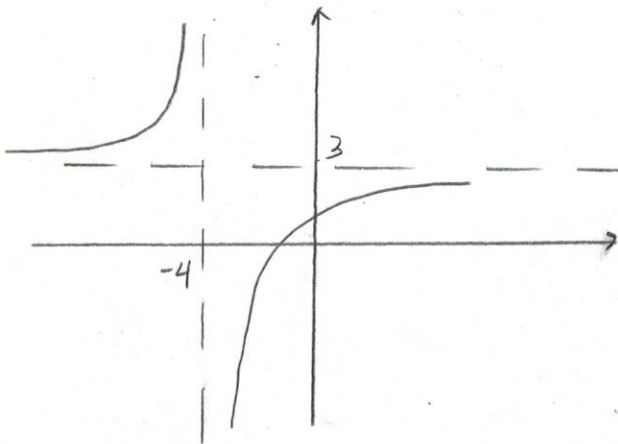
$53x = 265$

$x = 5 \text{ km}$

#14 $y = \frac{3x-5}{x+4}$

$\frac{3x-5}{x+4} = \frac{3x+12}{3} - \frac{17}{x+4}$

$y = \frac{-17}{x+4} + 3$



dom $f = \mathbb{R} \setminus \{-4\}$

Ima $f = \mathbb{R} \setminus \{3\}$

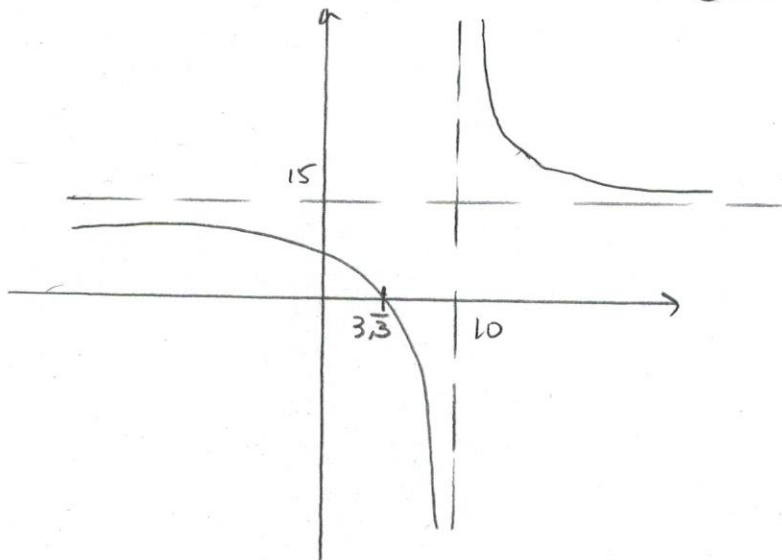
D

#15

$$f(x) = \frac{15x - 50}{x - 10}$$

$$\begin{array}{r} 15x - 50 \quad | \quad x - 10 \\ \underline{15x - 150} \quad | \quad 15 \\ 100 \end{array}$$

$$y = \frac{100}{x - 10} + 15$$



a) $\text{dom } f = \mathbb{R} \setminus \{10\}$

b) $\text{Im } f = \mathbb{R} \setminus \{15\}$

c) $0 = \frac{15x - 50}{x - 10}$

$$0 = 15x - 50$$

$$3,3 = x$$

d) Décroissante sur son domaine

e) $f(x) > 0 \quad]-\infty, 3,3[\cup]10, +\infty[$

$f(x) < 0 \quad]3,3, 10[$

f) $x = 10 \quad y = 15$

#16 a) $x = 0 \quad y = ? \quad Q = \frac{5(0) + 150}{0 + 5} = 30 \text{ milliards} = \text{qté initiale}$

1) $15 = \frac{5x + 150}{x + 5}$

$$15x + 75 = 5x + 150$$

$$10x = 75$$

$$x = 7,5 \text{ heures}$$

2) $7,5 = \frac{5x + 150}{x + 5}$

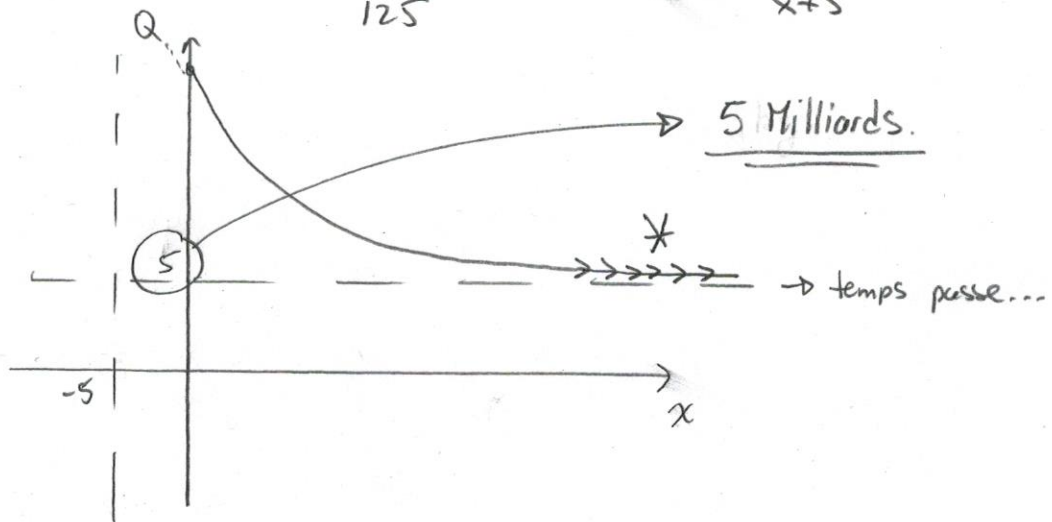
$$7,5x + 37,5 = 5x + 150$$

$$2,5x = 112,5$$

$$x = 45 \text{ heures}$$

b) Demandez :

#16b) Méthode 1: Graphique $\frac{5x+150}{5x+25} \cdot \frac{x+5}{5}$ $y = \frac{125}{x+5} + 5$



SANS DESSIN

Méthode 2: $\lim_{x \rightarrow +\infty} \frac{125}{x+5} + 5 = \frac{125}{+\infty + 5} + 5 = \frac{125}{+\infty} + 5 = 0 + 5 = 5$
5 Milliards

Méthode 3: $\lim_{x \rightarrow +\infty} \frac{5x+150}{x+5}$

$\lim_{x \rightarrow +\infty} \frac{x \cdot (5 + \frac{150}{x})}{x \cdot (1 + \frac{5}{x})} = \text{si } x \neq 0$

$\lim_{x \rightarrow +\infty} \frac{5 + \frac{150}{x}}{1 + \frac{5}{x}} = \frac{5 + \frac{150}{+\infty}}{1 + \frac{5}{+\infty}} = \frac{5+0}{1+0} = \underline{\underline{5 \text{ Milliards}}}$

$$\#17 \quad a) \quad |-5+3| = |-5|+|3| \quad (\text{F}) \\ |-2| = 5+3 \\ 2 \neq 8$$

$$b) \quad \left| \frac{-15}{3} \right| = \frac{|-15|}{|3|} \quad (\text{V}) \\ |-5| = \frac{15}{3} \\ 5 = 5$$

$$c) \quad |n-1| = |-n+1| \quad (\text{V}) \\ |n-1| = |-1(n-1)| \quad (\text{MES}) \\ |n-1| = |-1| |n-1| \\ |n-1| = 1 \cdot |n-1| \\ |n-1| = |n-1|$$

$$d) \quad -x = \ominus \quad (\text{F}) \quad \text{car si } x \text{ est négatif} \\ \text{alors } -x = \oplus$$

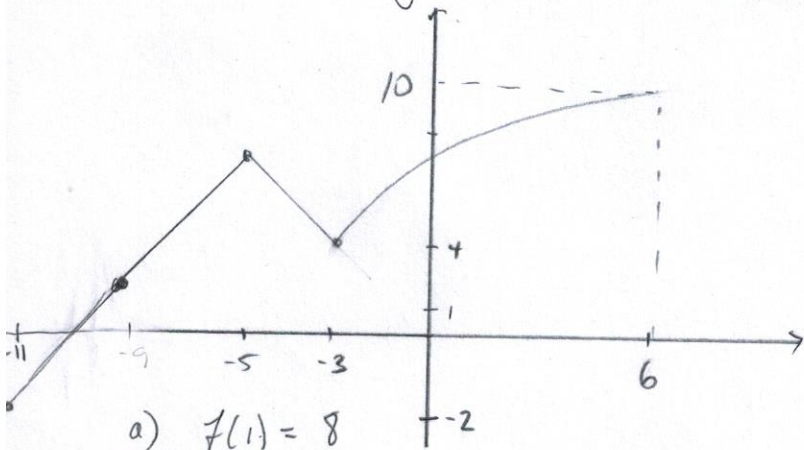
$$e) \quad |x| = \ominus \quad (\text{F})$$

$$\#18 \quad a) \quad a \ominus b \oplus \quad b) \quad a \ominus b \ominus \quad c) \quad a \oplus b \oplus \quad d) \quad a \oplus b \ominus$$

#19

Soit $f(x) = \begin{cases} -1,5|x+5|+7 & \text{si } -11 \leq x \leq -3 \\ 2\sqrt{x+3}+4 & \text{si } -3 \leq x \leq 6 \end{cases}$

e) Représenter graphiquement cette fonction



- $f(1) = 8$
- $f(-9) = 1$
- Quelle est la v.i. $x=0$ $y = 2\sqrt{3}+4 = 7,46\dots$
- dom $f = [-11, 6]$
- Sur quel intervalle est-elle positive?
- trouver le maximum $y=10$

d) Quel est le domaine $[-11, 6]$

b) Quelle est l'Image $[-2, 10]$

a) Quelle est la valeur de $f(1) \Rightarrow 8$

b) " " $f(-9) \Rightarrow 1$

#20

$$f(x) = \begin{cases} \sqrt{x+4} & x \in [-4, 0] \\ -2,5|x-4| + 12 & x \in [0, 10] \end{cases}$$

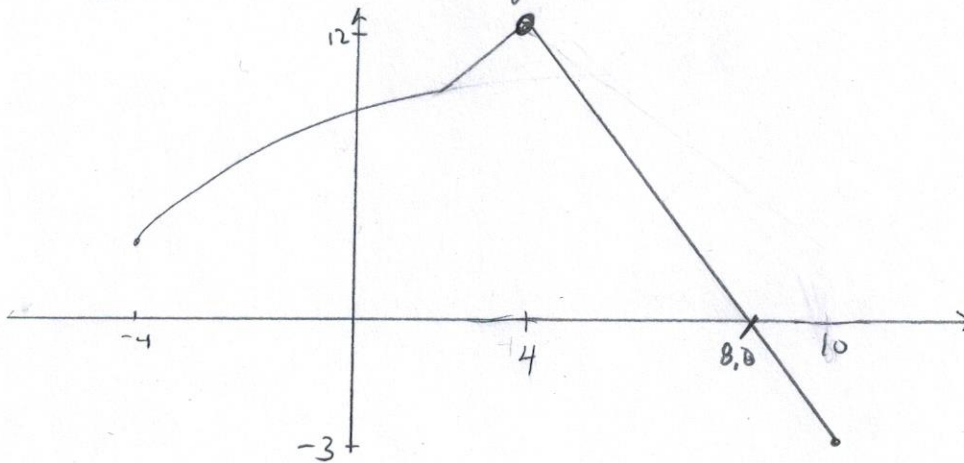
a) $f(-1) = ?$ $(\sqrt{3})$

b) $f(0) = ?$ (2)

c) $f(10) = ?$ (-3)

d) Dom $f = ?$ $([-4, 10])$

e) Tracez l'esquisse de cette fonction



f) Intervalle positive $[-4, 8,8]$

$$\hookrightarrow 0 = -2,5|x-4| + 12$$

$$4,8 = |x-4|$$

$$x \geq 4 \quad \wedge \quad x < 4$$

$$x-4 = 4,8 \quad x-4 = -4,8$$

$$\underline{x = 8,8} \quad x = 0,8$$

g) Quelle est l'image $[-3, 12]$.

#21

$$f(x) = \begin{cases} \frac{3}{x+1} + 3 & [-4, -1[\\ x+5 & [0, 7] \end{cases}$$

a) $f(-4) = ?$ (2)

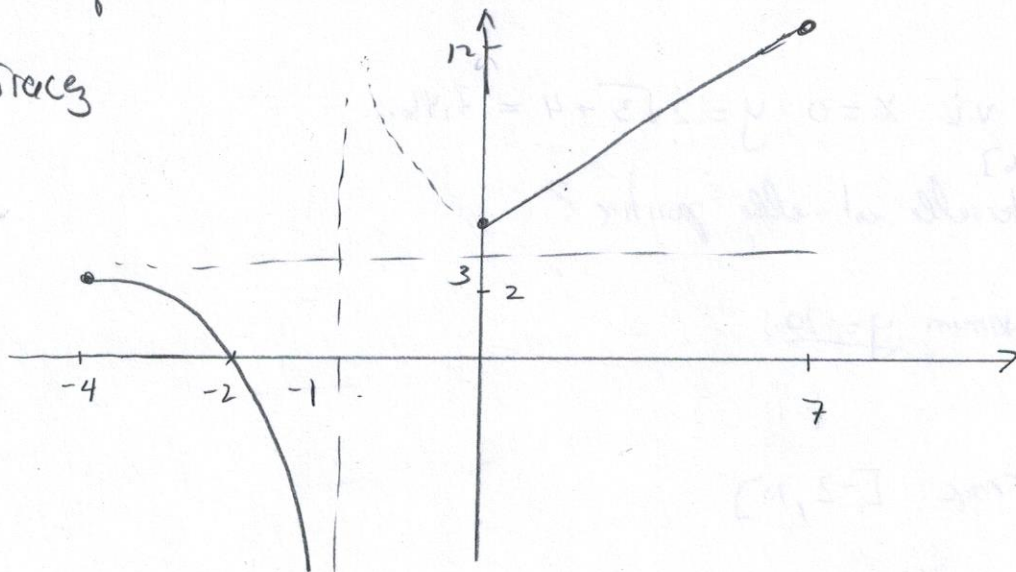
b) $f(6) = ?$ (11)

c) $f(0) = ?$ (5)

d) $f(-0.5) = ?$ ~~7~~

e) $\text{Dom } f = [-4, -1[\cup [0, 7]$

f) Traces



g) Intervalle \ominus $0 = \frac{3}{x+1} + 3$ $[-2, -1[$

$$-3(x+1) = 3$$

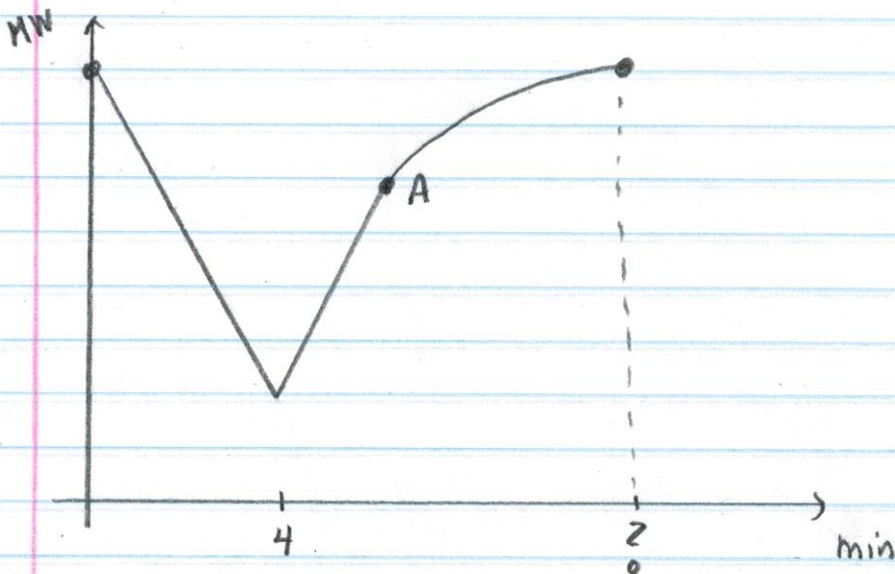
$$x+1 = -1$$

$$x = -2$$

h) Image $-\infty, 3[\cup [5, 12]$

$$\hookrightarrow f(0) = 5$$

#22



1) $y_1 = 4|x-4| + k_1, \therefore (5,8)$

$$8 = 4|5-4| + k_1$$
$$4 = k_1$$

2) $y_2 = 1,47\sqrt{x-6,5} + k_2 \quad k_2 = ?$

$$x_A = 6,5 \quad y_A = ? = k_2$$

$$\left. \begin{aligned} y_A &= 4|6,5-4| + 4 \\ y_A &= 10 + 4 = 14 \end{aligned} \right\} A(6,5; 14)$$

3) $x=0 \quad y=?$

$$y = 4|0-4| + 4$$

$$y = 4(4) + 4$$

$$y = 20$$

4) $x=? \quad \text{si } y=20$

$$20 = 1,47\sqrt{x-6,5} + 14$$

$$6 = 1,47\sqrt{x-6,5}$$

$$4,0816 = \sqrt{x-6,5} \quad x-6,5 \geq 0$$

$$16,6597... = x-6,5 \quad x \geq 6,5$$

$$23,16 = x$$

23,16 min