

Méchants dans problèmes : LES FONCTIONS TRIGONOMÉTRIQUES

#1 a) $\sin X = \frac{x}{z}$ b) $\cos Y = \frac{x}{z}$ c) $\tan X = \frac{x}{y}$ d) $\csc Y = \frac{1}{\sin Y} = \frac{1}{\frac{y}{z}} = \frac{z}{y}$

e) $\sec X = \frac{1}{\cos X} = \frac{1}{\frac{y}{z}} = \frac{z}{y}$ f) $\cot Y = \frac{1}{\tan Y} = \frac{1}{\frac{y}{x}} = \frac{x}{y}$

#2 a) $45^\circ = \frac{\pi}{4}$ rad, $90^\circ = \frac{\pi}{2}$ rad, $\frac{3\pi}{4}$ rad = 135° , π rad = 180° , $225^\circ = \frac{5\pi}{4}$ rad

$270^\circ = \frac{3\pi}{2}$ rad, $\frac{7\pi}{4}$ rad = 315°

b) $30^\circ = \frac{\pi}{6}$ rad, $\frac{\pi}{3}$ rad = 60° , $120^\circ = \frac{2\pi}{3}$ rad, $\frac{5\pi}{6}$ rad = 150° , $\frac{7\pi}{6}$ rad = 210° , $240^\circ = \frac{4\pi}{3}$ rad

$300^\circ = \frac{5\pi}{3}$ rad, $\frac{11\pi}{6}$ rad = 330°

#3 a) $350^\circ = x$ rad } $\frac{350\pi}{180} = \frac{35\pi}{18}$ rad b) $140^\circ = x$ rad } $\frac{140\pi}{180} = \frac{7\pi}{9}$ rad
 $180^\circ = \pi$ rad }

c) $70^\circ = \frac{7\pi}{18}$ rad car $70^\circ = \frac{1}{2}$ de $140^\circ = \frac{7\pi}{9}$ rad d) $-110^\circ = x$ rad } $\frac{-110\pi}{180} = \frac{-11\pi}{18}$ rad
 $180^\circ = \pi$ rad }

#4 a) $\frac{\pi}{6}$ rad = 30° b) $\frac{5\pi}{12}$ rad = $\frac{5 \times 180}{12} = 75^\circ$ c) -3π rad = $-3 \times 180 = -540^\circ$

d) 7 rad = $7 \times 57,3^\circ$ ou 7 rad = x° } $\frac{7 \times 180}{\pi} = 401,07^\circ$ e) $0,75$ rad = $\frac{0,75 \times 180}{\pi} = 42,97^\circ$

#5 a) $P\left(\frac{4\pi}{3}\right)$ $\frac{4\pi}{3}$ rad = 240° donc III b) $P\left(\frac{7\pi}{3}\right)$ $\frac{7\pi}{3}$ rad = $7 \times 60^\circ = 420^\circ$ donc $1,16$ tours

c) $P(-23)$ -23 rad = x tours } $x = \frac{-23}{2\pi} = -3,66$ tours donc II

d) $P(90)$ $\frac{90}{2\pi} = 14,32$ tours donc II

$$\#6 \quad a) P(0) = (1,0) \quad b) P\left(-\frac{\pi}{2}\right) = (0,-1) \quad c) P(\pi) = (-1,0) \quad d) P(-2\pi) = (1,0)$$

$$e) P\left(-\frac{3\pi}{2}\right) = (0,1) \quad f) P\left(\frac{29\pi}{6}\right) = P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \quad g) P\left(\frac{27\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$\hookrightarrow \frac{29\pi}{6} - 2\left(\frac{12\pi}{6}\right) = \boxed{\frac{5\pi}{6}}$

$\hookrightarrow 2\pi \text{ rad} = 1 \text{ tour}$

$\hookrightarrow \frac{27\pi}{4} - 3\left(\frac{8\pi}{4}\right) = \boxed{\frac{3\pi}{4}}$

$\hookrightarrow 2\pi \text{ rad} = 1 \text{ tour}$

$$h) P\left(-\frac{11\pi}{6}\right) = P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \quad i) P\left(\frac{25\pi}{2}\right) = (0,1)$$

$\hookrightarrow \frac{25\pi}{2} - 6\left(\frac{4\pi}{2}\right) = \boxed{\frac{\pi}{2}}$

$\hookrightarrow 2\pi \text{ rad} = 1 \text{ tour}$

$$\#7 \quad a) \sin \frac{\pi}{3} = \boxed{\frac{\sqrt{3}}{2}} \quad b) \sin \frac{5\pi}{6} = \boxed{\frac{1}{2}} \quad c) \cos \frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}} \quad d) \cos \frac{7\pi}{6} = \boxed{-\frac{\sqrt{3}}{2}}$$

$$e) \tan \frac{-\pi}{3} = \frac{\sin \frac{-\pi}{3}}{\cos \frac{-\pi}{3}} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2} \times \frac{2}{1} = \boxed{-\sqrt{3}}$$

$$f) \cot \frac{4\pi}{3} = \frac{\cos \frac{4\pi}{3}}{\sin \frac{4\pi}{3}} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-\frac{1}{2} \times -\frac{2}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = \boxed{\frac{1}{\sqrt{3}}} \text{ or } \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

$$g) \csc \frac{3\pi}{4} = \frac{1}{\sin \frac{3\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \boxed{\frac{2}{\sqrt{2}}} \text{ or } \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$$

$$h) \sec 3\pi = \frac{1}{\cos 3\pi} = \frac{1}{-1} = \boxed{-1}$$

$$\#8 \quad m \widehat{AB} = \frac{\pi}{5} \times 10 = \underline{2\pi \text{ cm}} \text{ or } \underline{6,28 \text{ cm}}$$

$$m \widehat{CD} = \frac{\pi}{5} \times 12,5 = \underline{7,85 \text{ cm}}$$

$$m \widehat{EF} = \frac{\pi}{5} \times 16,7 = \underline{10,49 \text{ cm}}$$

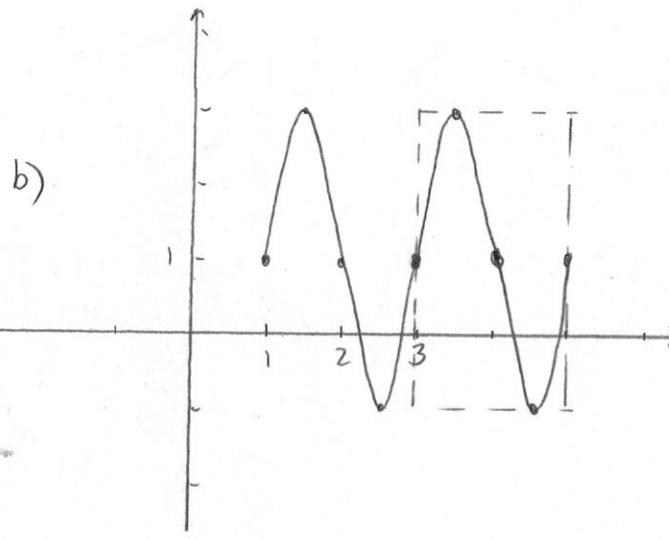
$$\#9 \quad \text{amplitude} = 3 \quad \text{periode} = 4$$

#10 a) $f(x) = 2 \sin \pi(x-3) + 1$

$$\begin{cases} \text{ampl.} = 2 \\ P = \frac{2\pi}{\pi} = 2 \end{cases}$$

$$(h, k) = (3, 1)$$

allgemein 

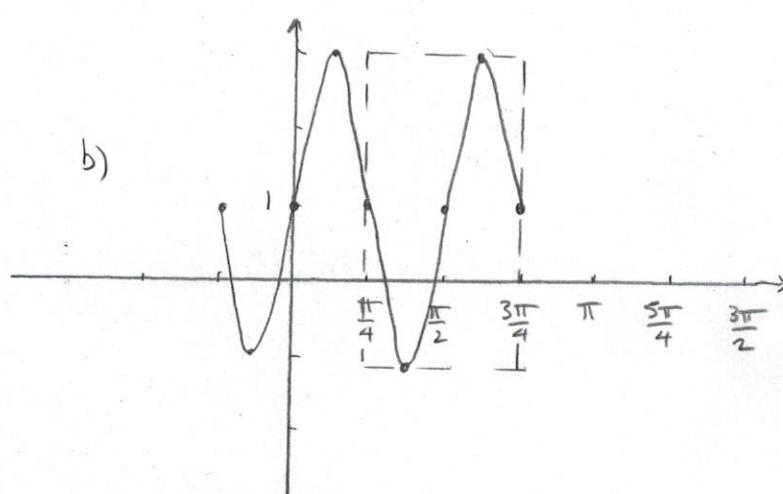


$g(x) = -2 \sin 4\left(x - \frac{\pi}{4}\right) + 1$

$$\begin{cases} \text{ampl.} = 2 \\ P = \frac{2\pi}{4} = \frac{\pi}{2} \end{cases}$$

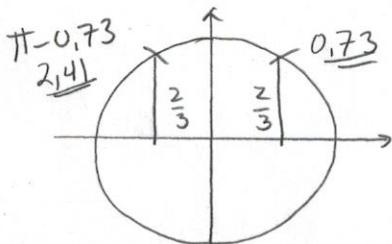
$$(h, k) = \left(\frac{\pi}{4}, 1\right)$$

allgemein 



#11 a) $0 = -3 \sin x + 2$

$$\frac{2}{3} = \sin x$$



$$\sin^{-1}\left(\frac{2}{3}\right) = 0,73 \text{ rad}$$

$$\boxed{\begin{aligned} x &= 0,73 + 2\pi n \\ x &= 2,41 + 2\pi n \end{aligned}} \quad | \quad n \in \mathbb{Z}$$

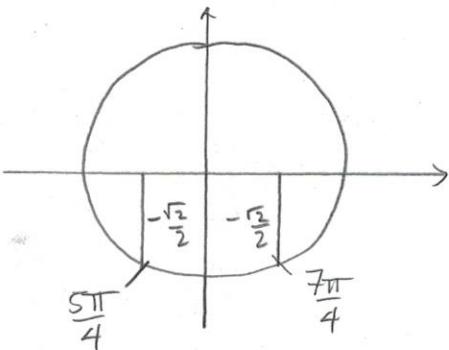
b) $0 = -\sin x + 3,5$

$$3,5 = \sin x$$

$\boxed{\emptyset}$ can $-1 \leq \sin x \leq 1$

$$11c) 0 = 2 \sin 2(x - \pi) + \sqrt{2}$$

$$-\frac{\sqrt{2}}{2} = \sin 2(x - \pi)$$

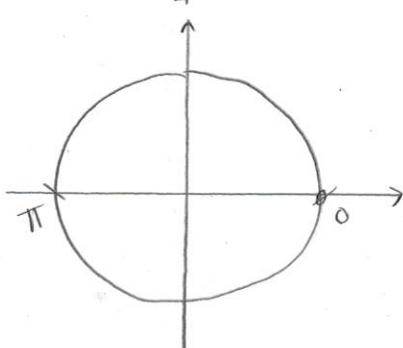


$$2(x - \pi) = \frac{5\pi}{4} \Rightarrow x - \pi = \frac{5\pi}{8} + \pi \Rightarrow x = \frac{13\pi}{8} + \pi n$$

$$2(x - \pi) = \frac{7\pi}{4} \Rightarrow x - \pi = \frac{7\pi}{8} + \pi \Rightarrow x = \frac{15\pi}{8} + \pi n$$

$$d) 0 = -4 \sin \frac{\pi}{4}(x+1)$$

$$0 = \sin \frac{\pi}{4}(x+1)$$

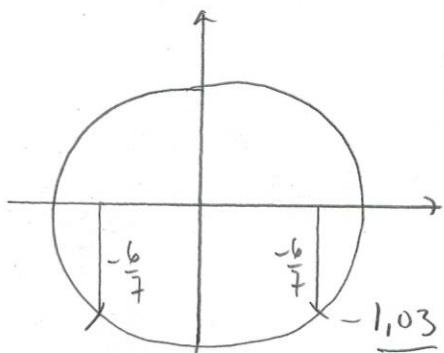


$$\frac{\pi}{4}(x+1) = 0 \Rightarrow x+1 = 0 \Rightarrow x = -1 + 8n$$

$$\frac{\pi}{4}(x+1) = \pi \Rightarrow x+1 = 4 \Rightarrow x = 3 + 8n$$

$$e) 0 = -7 \sin(x-1) + b$$

$$-\frac{b}{7} = \sin(x-1)$$



$$\sin^{-1}\left(-\frac{6}{7}\right) = -1,03 \text{ rad}$$

$$x-1 = -1,03 \Rightarrow x = -0,03 + 2\pi n$$

$$x-1 = 4,17 \Rightarrow x = 5,17 + 2\pi n$$

$$\pi - 1,03 \\ = 4,17$$

$$p = \frac{2\pi}{2} = \pi$$

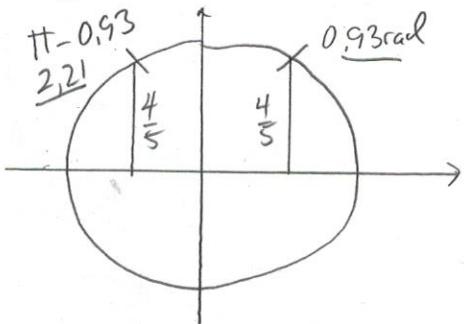
$$p = \frac{2\pi}{\frac{\pi}{4}} = 8$$

$$n \in \mathbb{Z}$$

$$n \in \mathbb{Z}$$

$$\#11 \text{ f) } 0 = -5 \sin(2x - 3) + 4$$

$$\frac{4}{5} = \sin(2x - 3)$$



$$\sin^{-1} \frac{4}{5} = 0,93 \text{ rad}$$

$$2x - 3 = 0,93 \Rightarrow$$

$$2x - 3 = 2,21 \Rightarrow$$

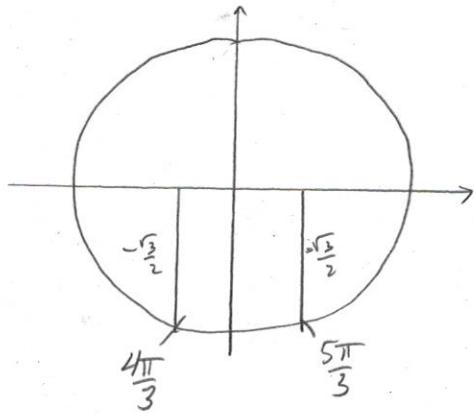
$$P = \frac{2\pi}{2} = \pi$$

$$\boxed{x = 1,97 + n\pi} \quad n \in \mathbb{Z}$$

$$\boxed{x = 2,61 + n\pi}$$

$$g) 0 = 2 \sin \frac{\pi}{2}(x-1) + \sqrt{3}$$

$$-\frac{\sqrt{3}}{2} = \sin \frac{\pi}{2}(x-1)$$



$$\frac{\pi}{2}(x-1) = \frac{4\pi}{3} \Rightarrow x-1 = \frac{4\pi}{3} \cdot \frac{2}{\pi} \Rightarrow x-1 = \frac{8}{3} \Rightarrow x = \frac{11}{3}$$

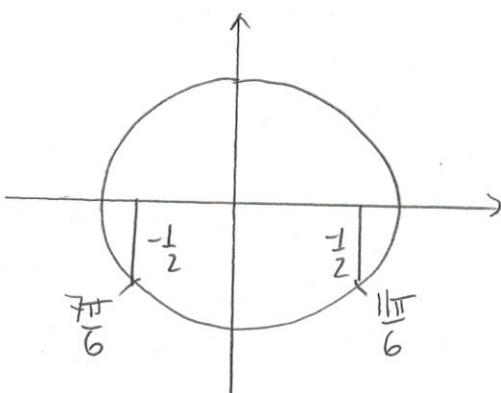
$$\frac{\pi}{2}(x-1) = \frac{5\pi}{3} \Rightarrow x-1 = \frac{5\pi}{3} \cdot \frac{2}{\pi} \Rightarrow x-1 = \frac{10}{3} \Rightarrow x = \frac{13}{3}$$

$$\boxed{\begin{array}{l} x = \frac{11}{3} + 4n \\ x = \frac{13}{3} + 4n \end{array}} \quad n \in \mathbb{Z}$$

$$P = \frac{2\pi}{\frac{\pi}{2}} = 4$$

$$h) 0 = 4 \sin 2(x - \frac{\pi}{7}) + 2$$

$$-\frac{1}{2} = \sin 2(x - \frac{\pi}{7})$$



$$2(x - \frac{\pi}{7}) = \frac{7\pi}{6} \Rightarrow x - \frac{\pi}{7} = \frac{7\pi}{12} \Rightarrow x = \frac{7\pi}{12} + \frac{\pi}{7} \rightarrow$$

$$2(x - \frac{\pi}{7}) = \frac{11\pi}{6} \Rightarrow x - \frac{\pi}{7} = \frac{11\pi}{12} \Rightarrow x = \frac{11\pi}{12} + \frac{\pi}{7} \rightarrow$$

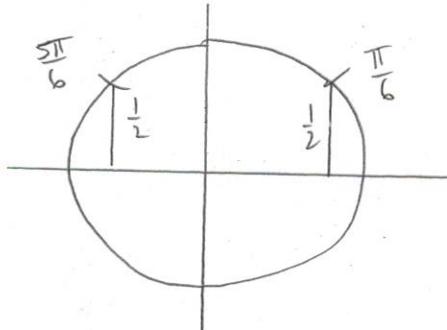
$$x = \frac{49\pi}{84} + \frac{12\pi}{84} = \boxed{\frac{61\pi}{84} + n\pi} \quad n \in \mathbb{Z}$$

$$x = \frac{77\pi}{84} + \frac{12\pi}{84} = \boxed{\frac{89\pi}{84} + n\pi} \quad n \in \mathbb{Z}$$

$$P = \frac{2\pi}{\frac{\pi}{2}} = \pi$$

$$\#11 \text{ i) } 0 = 6 \sin\left(\frac{\pi}{12}x - \frac{2\pi}{3}\right) - 3$$

$$\frac{1}{2} = \sin\left(\frac{\pi}{12}x - \frac{2\pi}{3}\right)$$



$$\frac{\pi}{12}x - \frac{2\pi}{3} = \frac{\pi}{6} \Rightarrow \frac{\pi}{12}x = \frac{\pi}{6} + \frac{2\pi}{3} \cdot \frac{2}{2} \Rightarrow \frac{\pi}{12}x = \frac{5\pi}{6}$$

$$\frac{\pi}{12}x + \frac{2\pi}{3} = \frac{5\pi}{6} \Rightarrow \frac{\pi}{12}x = \frac{5\pi}{6} - \frac{2\pi}{3} \cdot \frac{2}{2} \Rightarrow \frac{\pi}{12}x = \frac{9\pi}{6}$$

$$\frac{\pi}{12}x = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{6} \cdot \frac{12}{\pi} \Rightarrow x = 10 + 24n$$

$$\frac{\pi}{12}x = \frac{9\pi}{6} \Rightarrow x = \frac{9\pi}{6} \cdot \frac{12}{\pi} \Rightarrow x = 18 + 24n$$

$$p = \frac{2\pi}{\frac{\pi}{12}} = 24$$

#12 a) cycle partant de $(h, k) = (1, -1)$



$$y = 3 \sin \pi(x-1) - 1$$

$$p = 2 \text{ donc } 2 = \frac{2\pi}{b} \Rightarrow b = \pi$$

b) cycle partant de $(0, 1)$



$$y = -2 \sin 4x + 1$$

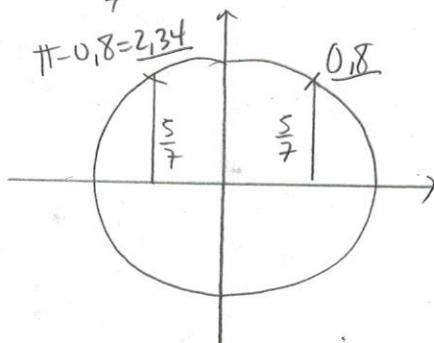
$$p = \frac{\pi}{2} \text{ donc } \frac{\pi}{2} = \frac{2\pi}{b} \Rightarrow b = 4$$

$$\#13 \text{ temps pour 1 tour complet} = 1 \text{ période} = \frac{2\pi}{b} = \frac{2\pi}{15} = 0,41887902 \text{ sec.}$$

$$\#14 \quad 25 = 35 \sin 0,06\pi t$$

$$\frac{25}{35} = \sin 0,06\pi t$$

$$\frac{5}{7} = \sin 0,06\pi t$$



$$\sin^{-1} \frac{5}{7} = 0,180$$

$$0,06\pi t = 0,18 \Rightarrow t = 4,24 + 33,3n$$

$$0,06\pi t = 2,34 \Rightarrow t = 12,41 + 33,3n$$

$$P = \frac{2\pi}{0,06\pi} = 33,3$$

a) $t_1 = 4,24 \text{ millisecondes}$

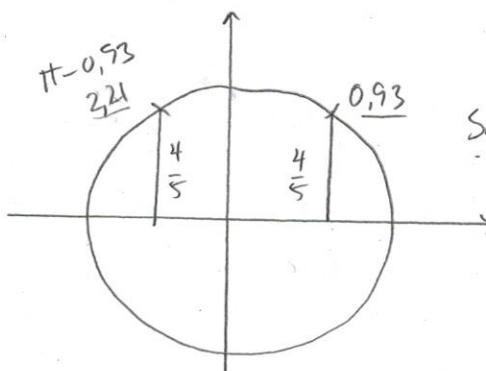
b) $4,24$, $37,57\bar{3}$, $70,90\bar{6}$
 $12,41$, $45,74\bar{3}$

#15

$$100 = 25 \sin \frac{\pi t}{12} + 80$$

$$20 = 25 \sin \frac{\pi t}{12}$$

$$\frac{4}{5} = \sin \frac{\pi}{12} t$$



$$\sin^{-1} \frac{4}{5} = 0,93$$

$$\frac{\pi}{12} t = 0,93 \Rightarrow t = 3,54 + 24n$$

n ∈ Z

$$\frac{\pi}{12} t = 2,21 \Rightarrow t = 8,46 + 24n$$

rep: $3,54^{\text{e}} \text{ mois}$, $27,54^{\text{e}} \text{ mois}$
 $8,46^{\text{e}} \text{ mois}$, $32,46^{\text{e}} \text{ mois}$

$$P = \frac{2\pi}{\frac{\pi}{12}} = 24$$

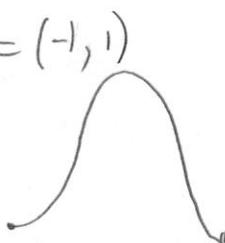
$$\#16 \quad f(x) = -2 \cos \frac{\pi}{2}(x+1) + 3$$

a) amplitude = 2

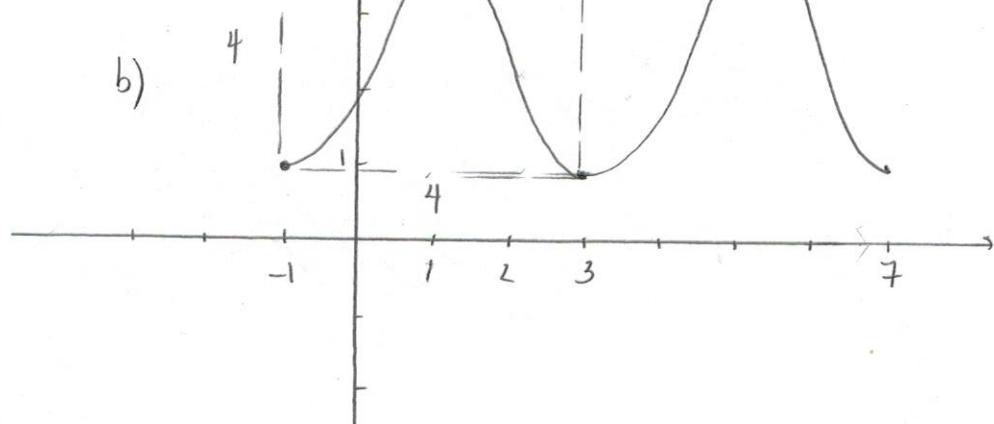
$$P = \frac{2\pi}{\frac{\pi}{2}} = 4$$

$$(h, k+a) = (-1, 1)$$

allure



b)



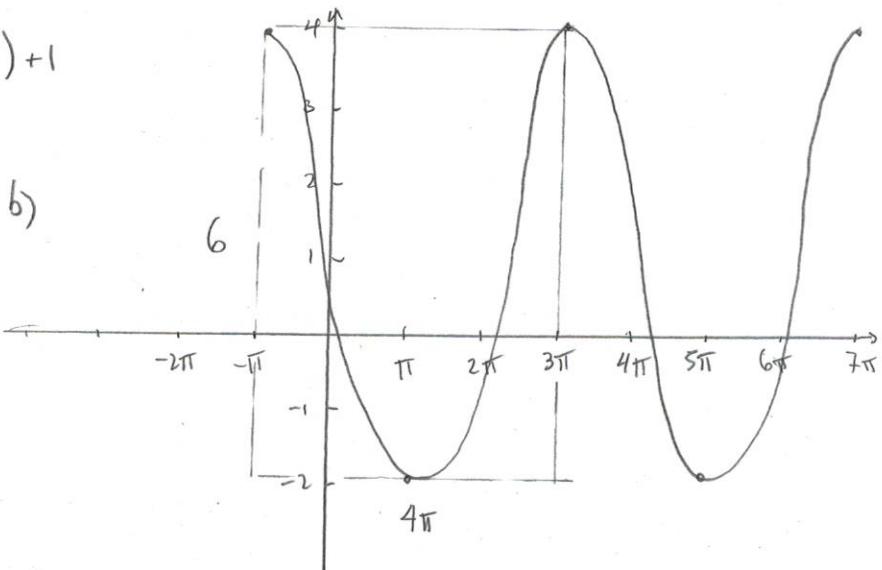
#16 site $y(x) = 3 \cos -\frac{1}{2}(x + \pi) + 1$

a) amplitude = 3

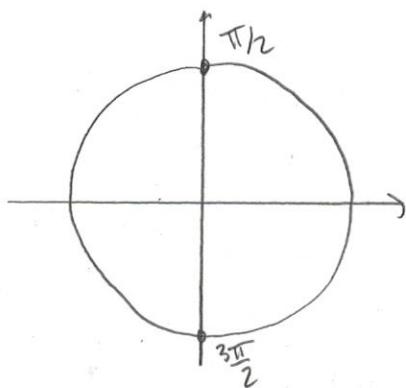
$$p = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

$$(h, k+a) = (-\pi, 4)$$

allure



#17 a) $0 = \cos 3(x - \pi)$



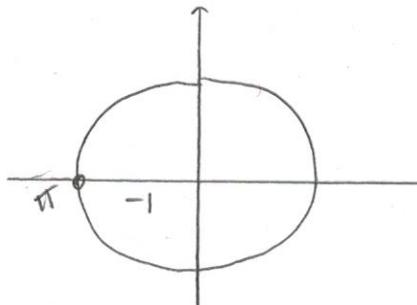
$$3(x - \pi) = \frac{\pi}{2} \Rightarrow x - \pi = \frac{\pi}{6} \Rightarrow x = \frac{7\pi}{6} + \frac{2\pi}{3}n$$

$\downarrow p = \frac{2\pi}{3}$
 $n \in \mathbb{Z}$

$$3(x - \pi) = \frac{3\pi}{2} \Rightarrow x - \pi = \frac{3\pi}{6} \Rightarrow x = \frac{9\pi}{6} = \frac{3\pi}{2} + \frac{2\pi}{3}n$$

b) $0 = \cos(x - 1) + 1$

$$-1 = \cos(x - 1)$$



$$x - 1 = \pi$$

$$\boxed{x = 4, 14 + 2\pi n} \quad n \in \mathbb{Z}$$

$$p = \frac{2\pi}{1} = 2\pi$$

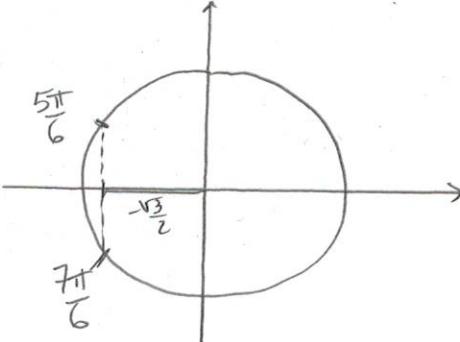
c) $0 = \cos x + 2$

$$-2 = \cos x$$

$$\boxed{\emptyset} \text{ can } -1 \leq \cos x \leq 1$$

$$\#17d) 0 = 2 \cos 2x + \sqrt{3}$$

$$-\frac{\sqrt{3}}{2} = \cos 2x$$

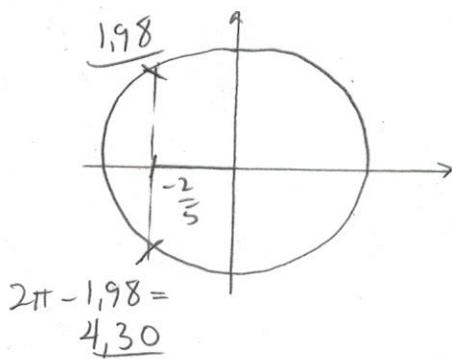


$$2x = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{12} + n\pi \quad P = \frac{2\pi}{2} = \pi$$

$$2x = \frac{7\pi}{6} \Rightarrow x = \frac{7\pi}{12} + n\pi \quad n \in \mathbb{Z}$$

$$e) 0 = 5 \cos(2x - 5) + 2$$

$$-\frac{2}{5} = \cos(2x - 5)$$



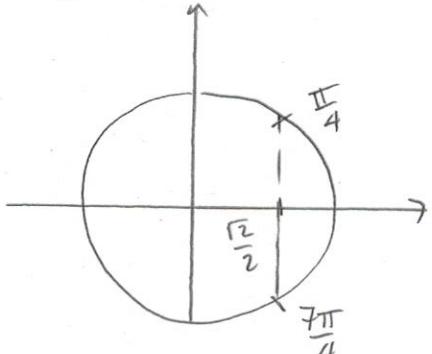
$$\cos^{-1}\left(-\frac{2}{5}\right) = 1.98$$

$$2x - 5 = 1.98 \Rightarrow x = 3.49 + n\pi \quad P = \frac{2\pi}{2} = \pi$$

$$2x - 5 = 4.30 \Rightarrow x = 4.165 + n\pi \quad n \in \mathbb{Z}$$

$$f) 0 = 2 \cos 2(x - \frac{\pi}{2}) - \sqrt{2}$$

$$\frac{\sqrt{2}}{2} = \cos 2(x - \frac{\pi}{2})$$



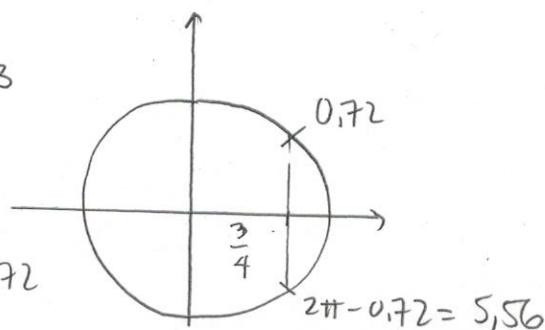
$$2(x - \frac{\pi}{2}) = \frac{\pi}{4} \Rightarrow x - \frac{\pi}{2} = \frac{\pi}{8} \Rightarrow x = \frac{5\pi}{8} + n\pi \quad P = \frac{2\pi}{2} = \pi$$

$$2(x - \frac{\pi}{2}) = \frac{7\pi}{4} \Rightarrow x - \frac{\pi}{2} = \frac{7\pi}{8} \Rightarrow x = \frac{11\pi}{8} + n\pi \quad n \in \mathbb{Z}$$

$$g) 0 = -4 \cos(x - 7) + 3$$

$$\frac{3}{4} = \cos(x - 7)$$

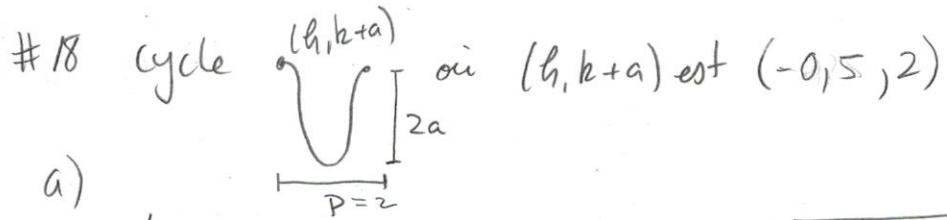
$$\cos^{-1}\left(\frac{3}{4}\right) = 0.72$$



$$x - 7 = 0.72 \Rightarrow x = 7.72 + 2\pi n$$

$$x - 7 = 5.56 \Rightarrow x = 12.72 + 2\pi n \quad n \in \mathbb{Z}$$

$$P = \frac{2\pi}{1}$$



a)

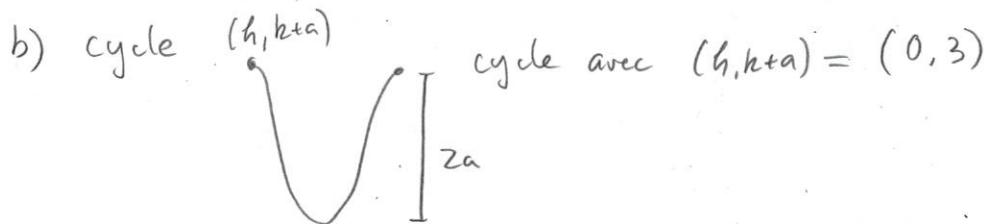
$$h = -0,5$$

$$k = \text{milieu} = -1$$

$$a = \text{demi-hauteur} = 3$$

$$p = 2 \text{ donc } p = 2 = \frac{2\pi}{b} \Rightarrow b = \pi$$

$$\boxed{y = 3 \cos \pi(x+0,5) - 1}$$



$$h = 0$$

$$k = \text{milieu} = 1$$

$$a = \text{demi-hauteur} = 2$$

$$p = \frac{\pi}{2} = \frac{2\pi}{b} \Rightarrow b = 4$$

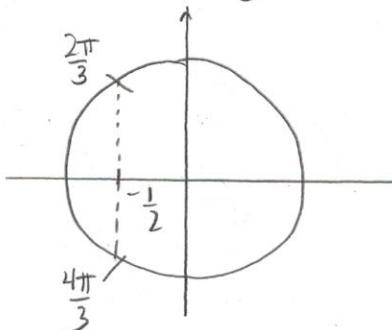
$$\boxed{y = 2 \cos 4x + 1}$$

#19

$$12 = -4 \cos \frac{2\pi}{3} t + 10$$

$$\frac{2}{-4} = \frac{-4 \cos \frac{2\pi}{3} t}{-4}$$

$$\frac{1}{2} = \cos \frac{2\pi}{3} t$$



$$\downarrow p = \frac{2\pi}{\frac{2\pi}{3}} = 3$$

$$\frac{2\pi}{3}t = \frac{2\pi}{3} \Rightarrow t = 1 + 3n$$

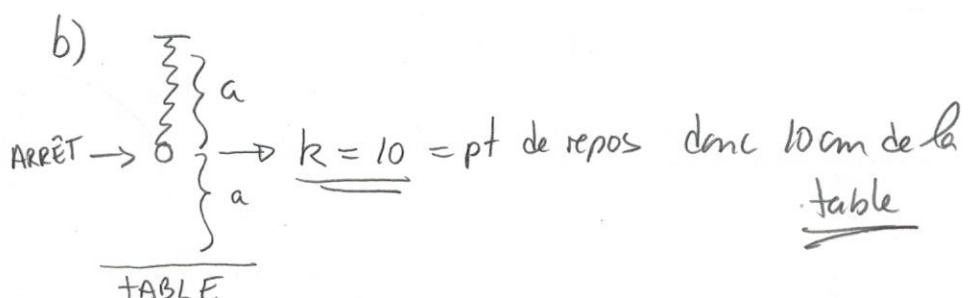
$$n \in \mathbb{Z}$$

$$\frac{2\pi}{3}t = \frac{4\pi}{3} \Rightarrow t = 2 + 3n$$

a) 1, 4, 7, 10, 13 sec

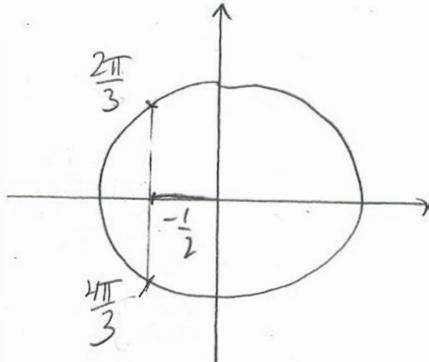
2, 5, 8, 11, 14 sec

$$\uparrow +3$$



$$\#20 \quad 0 = 250 \cos \frac{\pi}{15} t + 125$$

$$-\frac{1}{2} = \cos \frac{\pi}{15} t$$



$$\frac{\pi}{15} t = \frac{2\pi}{3} \Rightarrow t = \frac{2\pi}{3} \cdot \frac{15}{\pi} \Rightarrow t = 10 \text{ s}$$

$$\frac{\pi}{15} t = \frac{4\pi}{3} \Rightarrow t = \frac{4\pi}{3} \cdot \frac{15}{\pi} \Rightarrow t = 20 \text{ s}$$

10 sec

$$\#21 \quad h(x) = 18 \cos \frac{2\pi}{15} x + 95$$

$$\text{Max} = k+a = 95+18 = 113 \text{ cm}$$

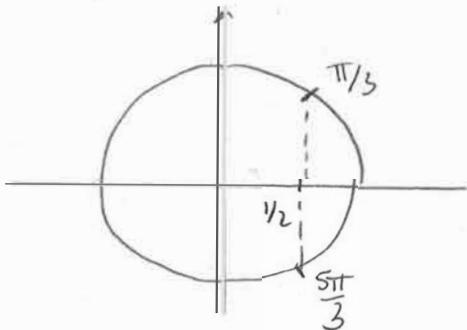
Réponse : 36 cm, Note: Nous aurions pu tout simplement trouver cet écart avec $2a = 36$

$$\text{Min} = k-a = 95-18 = 77 \text{ cm}$$

$$b) \quad P = \frac{2\pi}{b} = \frac{2\pi}{\frac{2\pi}{15}} = 15 \text{ sec}$$

$$c) \quad 104 = 18 \cos \frac{2\pi}{15} x + 95$$

$$\frac{1}{2} = \cos \frac{2\pi}{15} x$$



$$\frac{2\pi}{15} x = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{3} \cdot \frac{15}{2\pi} \Rightarrow x = 2,5 + 15n$$

$n \in \mathbb{Z}$

$$\frac{2\pi}{15} x = \frac{5\pi}{3} \Rightarrow x = \frac{5\pi}{3} \cdot \frac{15}{2\pi} \Rightarrow x = 12,5 + 15n$$

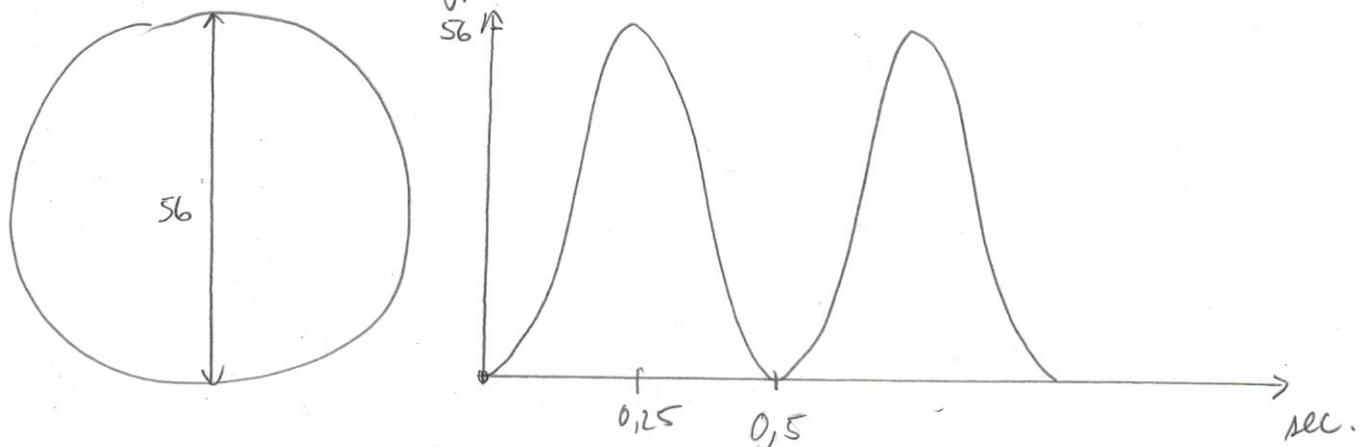
rep: $\boxed{2,5_n, 17,5_n, 32,5_n}$

$\boxed{12,5_n, 27,5_n}$

\downarrow

$+15$

#22



$$p = 0,5 \text{ sec} \text{ car } 0,25 \text{ sec} \rightarrow 0,5t$$

$x \leftarrow 1t$

$$y = a \cos b(x-h) + k$$

$$\text{ampl.} = \frac{1}{2} \cdot 56 = 28 \text{ donc } a = -28 \text{ car } \curvearrowright$$

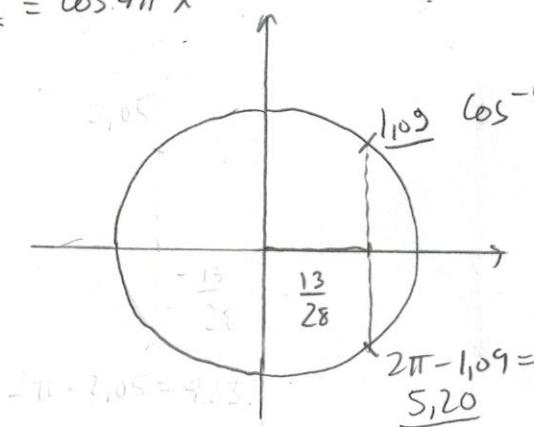
$$b = ? \quad p = 0,5 = \frac{2\pi}{b} \Rightarrow b = 4\pi$$

$$h = 0$$

$k = \text{milieu entre max et min donc } 28$

$$15 = -28 \cos 4\pi x + 28$$

$$\frac{13}{28} = \cos 4\pi x$$



$$\cos^{-1}\left(\frac{13}{28}\right) = 1,09$$

$$4\pi x = 1,09 \Rightarrow x = 0,087 + 0,5n$$

$$4\pi x = 5,20 \Rightarrow x = 0,413 + 0,5n$$

rép:

$0,087N$	$0,587N$
$0,413N$	$0,913N$

$+0,5$

#23

a) $\sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

b) $\arccos -1 = \pi$

c) $\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$

d) $\cos(\cos^{-1} 1) = \cos 0 = 1$

e) $\arcsin(\sin(\sin \frac{\pi}{2})) = \frac{\pi}{2}$

f) $\sin(\arccos \frac{1}{2}) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

g) $\arccos(\sin \frac{\pi}{6}) = \arccos \frac{1}{2}$

$= \frac{\pi}{3}$

h) $\sin(\cos^{-1}(-\frac{1}{2}))$

$= \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

i) $\cos(\arcsin \frac{\sqrt{2}}{2})$

$= \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

#24 a) $\tan^2 \theta + \cos^2 \theta + \cot^2 \theta = 1$

$\frac{\sin^2 \theta}{\cos^2 \theta} + \cos^2 \theta + \cot^2 \theta$

$\sin^2 \theta + \cos^2 \theta = 1 \quad \square$

b) $\sin^2 \theta + \cot^2 \theta + \sec \theta = \cos \theta$

$\sin^2 \theta + \frac{\cos^2 \theta}{\sin^2 \theta} + \frac{1}{\cos \theta} = \cos \theta$

$= \cos \theta \quad \square$

c) $\sec \theta - \cos \theta = \sin \theta \tan \theta$

$\frac{1}{\cos \theta} - \frac{\cos \theta}{1}$

$\frac{1}{\cos \theta} - \frac{\cos^2 \theta}{\cos \theta}$

$\frac{1 - \cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta} = \sin \theta \cdot \frac{\sin \theta}{\cos \theta} = \sin \theta \tan \theta \quad \square$

d) $(1 - \cos^2 \theta)(1 + \tan^2 \theta) = \tan^2 \theta$

$\sin^2 \theta \cdot \sec^2 \theta$

$\sin^2 \theta \cdot \frac{1}{\cos^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta \quad \square$

e) $(1 + \tan^2 \theta)(1 - \sin^2 \theta) = 1$

$\sec^2 \theta \cdot \cos^2 \theta$

$1 - \frac{1}{\cos^2 \theta} \cos^2 \theta = 1 \quad \square$

$= \sec^2 \theta - 1 \quad \square$

$$\#24f) (\sec \theta - \tan \theta)^2 = \frac{1 - \sin \theta}{1 + \sin \theta}$$

$$\left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right)^2$$

$$\left(\frac{1 - \sin \theta}{\cos \theta} \right)^2 = \frac{(1 - \sin \theta)(1 - \sin \theta)}{\cos^2 \theta} = \frac{(1 - \sin \theta)(1 - \sin \theta)}{1 - \sin^2 \theta}$$

$$= \frac{(1 - \sin \theta)(1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)}$$

difference & cosine

$$g) 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1 \quad \square$$

$$\begin{aligned} 1 - 2(1 - \cos^2 \theta) \\ = 1 - 2 + 2 \cos^2 \theta \\ = 2 \cos^2 \theta - 1 \end{aligned} \quad \square$$

$$h) \tan \theta + \cot \theta = \sec \theta \csc \theta$$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta}{\cos \theta \sin \theta} + \frac{\cos^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta} = \sec \theta \csc \theta \quad \square$$

$$\#25a) \tan^2 \theta + \sin^2 \theta - \tan^2 \theta \cos^2 \theta = \tan^2 \theta$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \sin^2 \theta - \frac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \sin^2 \theta - \sin^2 \theta$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta \quad \square$$

$$\#25 \text{ b) } \frac{\sin \theta \sec \theta \cot \theta}{\csc \theta} = \sin \theta$$

$$\frac{\cancel{\sin \theta} \cdot \frac{1}{\cancel{\cos \theta}} \cdot \frac{\cancel{\cos \theta}}{\sin \theta}}{\frac{1}{\sin \theta}} = \sin \theta \quad \square$$

$$\text{c) } \frac{\cos \theta + \sin^2 \theta}{1 - \cos \theta} = \sec \theta$$

$$\frac{\cancel{\cos \theta} + \cancel{\sin^2 \theta}}{\cancel{\cos \theta} - \cos \theta} = \sec \theta \quad \square$$

$$\text{e) } \sin \theta + \cot \theta \cos \theta = \csc \theta$$

$$\begin{aligned} & \sin \theta + \frac{\cos \theta}{\sin \theta} \cos \theta \\ &= \sin \theta + \frac{\cos^2 \theta}{\sin \theta} \end{aligned}$$

$$\begin{aligned} &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} \\ &= \frac{1}{\sin \theta} = \csc \theta \quad \square \end{aligned}$$

$$\#26 \text{ a) } \tan \theta (\sin \theta + \cot \theta \cos \theta) = \sec \theta$$

$$\begin{aligned} &= \tan \theta \sin \theta + \tan \theta \cot \theta \cos \theta \\ &= \frac{\sin \theta}{\cos \theta} \cdot \sin \theta + \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} \cdot \cos \theta \\ &= \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos \theta}{\cos \theta} \\ &= \frac{1}{\cos \theta} = \sec \theta \quad \square \end{aligned}$$

$$\text{b) } \sin \theta + \cot \theta \csc \theta = \sec \theta$$

$$\begin{aligned} & \sin \theta + \frac{\cos \theta}{\sin \theta} \frac{\cos \theta}{\sin \theta} \\ & \sin \theta + \frac{\cos^2 \theta}{\sin^2 \theta} \\ & \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} \\ & \frac{1}{\sin^2 \theta} = \sec \theta \quad \square \end{aligned}$$

$$\#26c) (\sec \theta + \tan \theta - 1)(\sec \theta - \tan \theta + 1) = 2 \tan \theta$$

$$\sec^2 \theta - \sec \theta \tan \theta + \cancel{\sec \theta} + \cancel{\sec \theta} \tan \theta - \tan^2 \theta + \tan \theta - \cancel{\sec \theta} + \tan \theta - 1$$

$$\sec^2 \theta - \tan^2 \theta + 2 \tan \theta - 1$$

$$\cancel{\tan^2 \theta} + 1 - \cancel{\tan^2 \theta} + 2 \tan \theta - 1 \quad \square$$

$$d) \frac{\cos^2 \theta}{1 - \sin \theta} = 1 + \sin \theta$$

$$\frac{1 - \sin^2 \theta}{1 - \sin \theta} = \frac{(1 + \sin \theta)(1 - \sin \theta)}{1 - \sin \theta} \quad \square \quad \text{Différence de carrés.}$$

$$e) \frac{1 + \tan^2 \theta}{\csc^2 \theta} = \tan^2 \theta$$

$$\frac{\sec^2 \theta}{\csc^2 \theta} = \frac{\frac{1}{\cos^2 \theta}}{\frac{1}{\sin^2 \theta}} = \frac{1}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{1} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta \quad \square$$

$$f) \frac{\sin \theta}{\sin \theta + \cos \theta} = \frac{\tan \theta}{1 + \tan \theta}$$

On démontre par le membre de droite ?

$$\frac{\frac{\sin \theta}{\cos \theta}}{1 + \frac{\sin \theta}{\cos \theta}} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\cos \theta + \sin \theta}{\cos \theta}} = \frac{\frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\cos \theta + \sin \theta}}{\cos \theta} = \frac{\sin \theta}{\cos \theta + \sin \theta} \quad \square$$

$$g) \frac{\cos \theta}{1 + \sin \theta} + \frac{\cos \theta}{1 - \sin \theta} = 2 \sec \theta$$

$$\frac{\cos \theta (1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)} + \frac{\cos \theta (1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}$$

$$\begin{aligned} \frac{\cos \theta - \cos \theta \sin \theta}{1 - \sin^2 \theta} + \frac{\cos \theta + \cos \theta \sin \theta}{1 - \sin^2 \theta} &= \frac{\cos \theta - \cos \theta \sin \theta + \cos \theta + \cos \theta \sin \theta}{1 - \sin^2 \theta} \\ &= \frac{2 \cos \theta}{\cos^2 \theta} = \frac{2}{\cos \theta} = 2 \sec \theta \quad \square \end{aligned}$$

$$\#26h) \sec \theta - \cos \theta = \sin \theta \tan \theta$$

$$\frac{1}{\cos \theta} - \cos \theta$$

$$\frac{1}{\cos \theta} - \frac{\cos^2 \theta}{\cos \theta}$$

$$\frac{1 - \cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta} = \sin \theta \cdot \frac{\sin \theta}{\cos \theta} = \sin \theta \tan \theta \quad \square$$

$$\#27 a) \frac{\tan^2 \theta}{1 + \tan^2 \theta} \cdot \frac{1 + \cot^2 \theta}{\cot^2 \theta} = \sin^2 \theta \sec^2 \theta$$

$$\begin{aligned} \frac{\sin^2 \theta}{\sec^2 \theta} \cdot \frac{\csc^2 \theta}{\frac{\cos^2 \theta}{\sin^2 \theta}} &= \frac{\sin^2 \theta}{\sec^2 \theta} \cdot \frac{1}{\frac{1}{\sec^2 \theta}} \\ &= \frac{\sin^2 \theta}{\sec^2 \theta} \cdot \frac{1}{\frac{1}{\sin^2 \theta}} \cdot \frac{\sin^2 \theta}{\sec^2 \theta} = \sin^2 \theta \cdot \frac{1}{\sec^2 \theta} \\ &= \sin^2 \theta \cdot \sec^2 \theta \quad \square \end{aligned}$$

$$b) (\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2 = \tan^2 \theta + \cot^2 \theta + 7$$

$$\underline{\sin^2 \theta} + 2 \sin \theta \csc \theta + \csc^2 \theta + \underline{\cos^2 \theta} + 2 \cos \theta \sec \theta + \sec^2 \theta$$

$$\underbrace{\sin^2 \theta + \cos^2 \theta}_{1} + 2 \sin \theta \cdot \frac{1}{\sin \theta} + 2 \cos \theta \cdot \frac{1}{\cos \theta} + \csc^2 \theta + \sec^2 \theta$$

$$\underbrace{1 + 2 \sin \theta \cdot \frac{1}{\sin \theta} + 2 \cos \theta \cdot \frac{1}{\cos \theta}}_{5 + \csc^2 \theta + \sec^2 \theta} + \csc^2 \theta + \sec^2 \theta$$

$$\frac{5 + \csc^2 \theta + \sec^2 \theta}{5 + \cot^2 \theta + 1 + \tan^2 \theta + 1}$$

$$\cot^2 \theta + \tan^2 \theta + 7 \quad \square$$

$$c) \sec^4 \theta - 1 = 2 \tan^2 \theta + \tan^4 \theta$$

$$\underbrace{(\sec^2 \theta - 1)}_{(\sec^2 \theta - 1)(\sec^2 \theta + 1)} : \text{Diff. de carrés.}$$

$$\tan^2 \theta (\sec^2 \theta + 1)$$

$$\tan^2 \theta (\tan^2 \theta + 1 + 1)$$

$$\tan^2 \theta (\tan^2 \theta + 2)$$

$$\Rightarrow \tan^4 \theta + 2 \tan^2 \theta \quad \square$$

$$\#27d) \sin^4 \theta - \cos^4 \theta = 1 - 2\cos^2 \theta$$

$(\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)$: différence de carrés.

$$1 \cdot (\overbrace{1 - \cos^2 \theta}^{\downarrow} - \cos^2 \theta) = 1 - 2\cos^2 \theta \quad \square$$

$$e) (1 + \tan^2 \theta)^2 + (1 - \tan^2 \theta)^2 = 2 \sec^2 \theta$$

$$1 + 2\tan \theta + \tan^2 \theta + 1 - 2\tan \theta + \tan^2 \theta$$

$$2 + 2\tan^2 \theta$$

$$2 + 2(\sec^2 \theta - 1) \quad \text{car } 1 + \tan^2 \theta = \sec^2 \theta$$

$$2 + 2\sec^2 \theta - 2$$

$$2\sec^2 \theta \quad \square$$

$$f) \sin^2 \theta \underbrace{(1 + \cos^2 \theta)}_{\downarrow} + \cos^2 \theta \underbrace{(1 + \tan^2 \theta)}_{\downarrow} = 2$$

$$\sin^2 \theta \cdot \csc^2 \theta + \cos^2 \theta \cdot \sec^2 \theta$$

$$\sin^2 \theta \cdot \frac{1}{\sin^2 \theta} + \cos^2 \theta \cdot \frac{1}{\cos^2 \theta}$$

$$1 + 1 = 2 \quad \square$$

$$g) (1 - \sin \theta + \cos \theta)^2 = 2(1 - \sin \theta)(1 + \cos \theta)$$

$$(1 - \sin \theta + \cos \theta)(1 - \sin \theta + \cos \theta)$$

$$1 - \underline{\sin \theta} + \underline{\cos \theta} - \underline{\sin \theta} + \sin^2 \theta - \underline{\sin \theta \cos \theta} + \underline{\cos \theta} - \underline{\sin \theta \cos \theta} + \cos^2 \theta$$

$$\equiv 1 - 2\sin \theta + 2\cos \theta + \cancel{\sin^2 \theta + \cos^2 \theta} - \cancel{2\sin \theta \cos \theta}$$

$$2 - 2\sin \theta + 2\cos \theta - 2\sin \theta \cos \theta$$

$$2(1 - \sin \theta + \cos \theta - \sin \theta \cos \theta)$$

$$2(1 - \sin \theta + \cos \theta(1 - \sin \theta)) \quad \text{MES de } \cos \theta$$

$$2[(1 - \sin \theta)(1 + \cos \theta)] \quad \text{MES de } (1 - \sin \theta)$$

\square

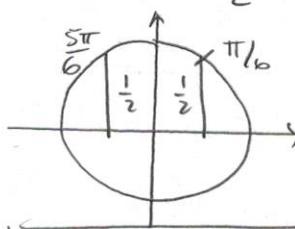
$$\#27h) \frac{\sec^2 \theta \cdot \cot \theta}{\csc^2 \theta} = \tan \theta$$

$$\frac{\frac{1}{\cos^2 \theta} \cdot \frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin^2 \theta}} = \frac{\frac{1}{\cos \theta \sin \theta}}{\frac{1}{\sin^2 \theta}} = \frac{1}{\cos \theta \sin \theta} \cdot \frac{\sin^2 \theta}{1} = \frac{\sin \theta}{\cos \theta} = \tan \theta \quad \square$$

$$\#28a) (2\sin x - 1)(\sin x + 0.5) = 0$$

$$2\sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

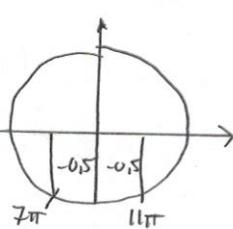


$$x = \frac{5\pi}{6} + 2\pi n$$

$$x = \frac{\pi}{6} + 2\pi n \quad n \in \mathbb{Z}$$

$$\sin x + 0.5 = 0$$

$$\sin x = -0.5$$

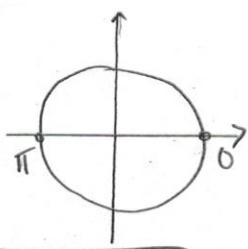


$$x = \frac{7\pi}{6} + 2\pi n$$

$$x = \frac{11\pi}{6} + 2\pi n \quad n \in \mathbb{Z}$$

$$b) \sin x (3\sin x - 2) = 0$$

$$\sin x = 0$$

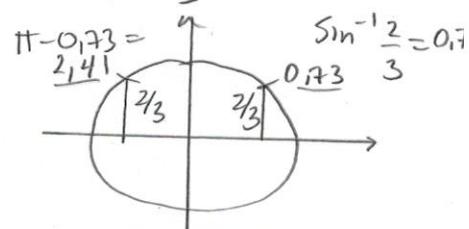


$$x = 0 + 2\pi n$$

$$x = \pi + 2\pi n$$

$$3\sin x - 2 = 0$$

$$\sin x = \frac{2}{3}$$



$$x = 0.73 + 2\pi n$$

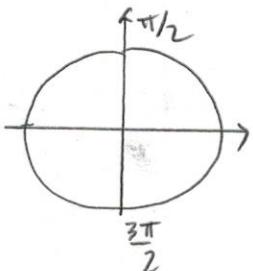
$$x = 2.41 + 2\pi n \quad n \in \mathbb{Z}$$

$$c) \cos x \sin x = -\cos x$$

$$\cos x \sin x + \cos x = 0$$

$$\cos x (\sin x + 1) = 0$$

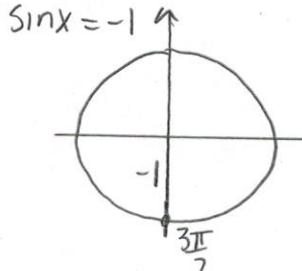
$$\cos x = 0$$



$$x = \frac{\pi}{2} + 2\pi n$$

$$x = \frac{3\pi}{2} + 2\pi n$$

$$\sin x + 1 = 0$$



$$x = \frac{3\pi}{2} + 2\pi n$$

$$n \in \mathbb{Z}$$

$$d) 2\sec x = \cos x + 1$$

$$\frac{2 \cdot \frac{1}{\cos x}}{\cos x} = \cos x + 1 \cdot \frac{\cos x}{\cos x}$$

$$2 = \cos^2 x + \cos x$$

$$0 = \cos^2 x + \cos x - 2$$

$$0 = (\cos x + 2)(\cos x - 1)$$

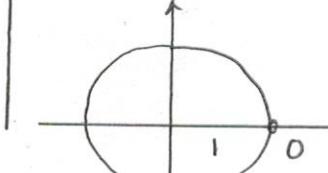
$$\cos x + 2 = 0$$

$$\cos x = -2$$

\emptyset

$$\cos x - 1 = 0$$

$$\cos x = 1$$



$$x = 0 + 2\pi n$$

$$n \in \mathbb{Z}$$

$$\# 28 \text{ e}) \quad \tan x + 3 \cot x = 4$$

$$\tan x + \frac{3}{\tan x} = 4$$

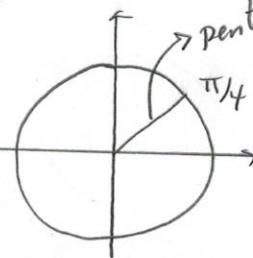
$$\tan^2 x + 3 = 4 \tan x$$

$$\tan^2 x - 4 \tan x + 3 = 0$$

$$(\tan x - 1)(\tan x - 3) = 0$$

$$\tan x - 1 = 0$$

$$\tan x = 1$$

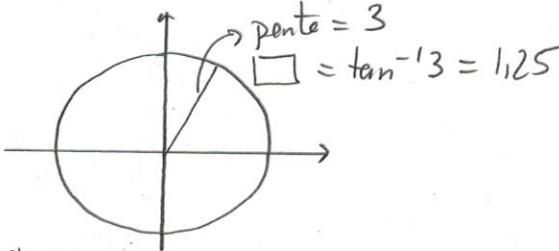


$$x = \frac{\pi}{4} + \pi n$$

$$n \in \mathbb{Z}$$

$$\tan x - 3 = 0$$

$$\tan x = 3$$



$$x = 1,25 + \pi n$$

$$f) \quad 2 - 2 \sin^2 x = 11 \cos x - 5$$

$$2 - 2(1 - \cos^2 x) = 11 \cos x - 5$$

$$2 - 2 + 2 \cos^2 x = 11 \cos x - 5$$

Methode S-P

$$2 \cos^2 x - 11 \cos x + 5 = 0 \rightarrow P: 10 \quad \boxed{-1} \quad \boxed{-10}$$

$$S: -11$$

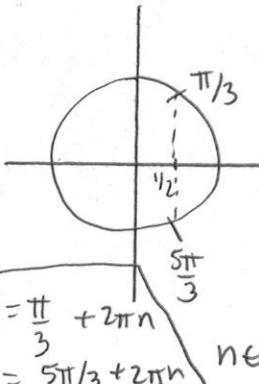
$$2 \cos^2 x - 6 \cos x - 10 \cos x + 5 = 0$$

$$\cos x (2 \cos x - 1) - 5 (2 \cos x - 1) = 0$$

$$(2 \cos x - 1)(\cos x - 5) = 0$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$



$$x = \frac{\pi}{3} + 2\pi n$$

$$x = \frac{5\pi}{3} + 2\pi n$$

$$\cos x - 5 = 0$$

$$\cos x = 5$$

\emptyset

$$n \in \mathbb{Z}$$

$$\#28 \text{ g}) \quad \cot x - 5 \csc x + 3 \tan x = 0$$

$$\frac{\cos x}{\sin x} - \frac{5}{\sin x} + 3 \frac{\sin x}{\cos x} = 0$$

$$\frac{\cos^2 x}{\sin x \cos x} - \frac{5 \cos x}{\sin x \cos x} + \frac{3 \sin^2 x}{\cos x \sin x} = 0$$

$$\cos^2 x - 5 \cos x + 3 \sin^2 x = 0$$

$$\cos^2 x - 5 \cos x + 3(1 - \cos^2 x) = 0$$

$$\cos^2 x - 5 \cos x + 3 - 3 \cos^2 x = 0$$

$$-2 \cos^2 x - 5 \cos x + 3 = 0 \quad \text{Methode S-P}$$

$$-2 \cos^2 x - 6 \cos x + \cos x + 3 = 0$$

$$-2 \cos x (\cos x + 3) + 1 (\cos x + 3) = 0$$

$$\begin{array}{l} P = -6 \\ S = -5 \end{array} \quad \boxed{-6} \quad \boxed{1}$$

$$(\cos x + 3)(-2 \cos x + 1) = 0$$

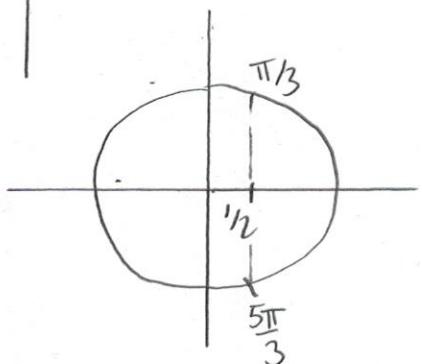
$$\cos x + 3 = 0$$

$$\cos x = -3$$

\emptyset

$$-2 \cos x + 1 = 0$$

$$\cos x = \frac{1}{2}$$



$$\begin{cases} x = \frac{\pi}{3} + 2\pi n \\ x = \frac{5\pi}{3} + 2\pi n \end{cases}$$

$n \in \mathbb{Z}$